BCQM Primitives and the Emergence of Spacetime

(Foundational Note v1.0)

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Abstract

We propose a minimal, amplitude-first primitive for Boundary-Condition Quantum Mechanics (BCQM) in which (i) primitives are events, directed edges, and complex edge amplitudes; (ii) a single hop-bounded, locally windowed selection rule chooses the next realized event with probability $\propto |K_r|^2$; and (iii) two scales anchor experiments: a short-window envelope σ and an empirical coherence horizon W_{coh}. We give operator-theoretic bounds (Neumann-series control and LLN/CLT under local boundedness), show no-signalling is preserved, and motivate the envelope from GKLS-style local dephasing. To encode records, inertia-like drift, and curvature from inhomogeneities, we introduce a strictly local back-reaction (Axiom 6) that remodels edge amplitudes near each realized event; we provide concrete, equivariant examples and a small menu of remodeling maps. An appendix outlines simulations with falsifiable hooks (e.g., a PSD knee $\sim 1/W_{\text{coh}}$, Zeno scaling as $\sigma \downarrow 0$, and anisotropy self-averaging). The note is intended as a precise, testable proposal; continuum limits and full gravitational dynamics are deferred to follow-ups.

Remark 1 (Quick map for the reader)**.** ***Primitives.*** *Events (nodes), directed connections (edges), and complex edge amplitudes a.*
One rule. *From the last realized event E_n, pick the next event x with probability $\propto \left|K_r(E_n \to x)\right|^2$, where K_r is a hop-bounded, locally windowed sum of path amplitudes $A[\gamma]$ with envelope $F[\gamma;\sigma]$.*
What emerges. *In the dense limit, the web of realized events admits a stable coarse-grained geometry; the coherence horizon W_{coh} sets a floor on recoverable interference.*

This box is a non-technical compass; formal definitions follow immediately.

Remark 2 (Scope of this note)**.** *We provide precise primitives, a single stochastic selection rule, operator-theoretic bounds sufficient for truncated kernels, and empirical hooks via the coherence horizon W_{coh}. Full continuum limits, uniqueness of 3+1 emergence, and dynamical gravity/inertia are deferred; open problems are flagged explicitly and simulations are outlined in the Appendix.*

Remark 3 (Notation convention: W vs. w)**.** *Throughout, W denotes the* physical coherence horizon *(BCQM I–III). The primitive step uses a short-hop/window parameter w in $F[\gamma; w]$, chosen with $w \lesssim W_{\text{coh}}$. This keeps the physical horizon (W) distinct from the local envelope used in the hop-bounded kernel (w) and from the separate envelope parameter (σ) used for exponential damping in the operator-theoretic core.* [1]

[1] Here the local window w is chosen $\lesssim W_{\text{coh}}$ for consistency with Paper III.

Purpose

This note records the *primitive, pre-geometric rules* of BCQM and a high-level, technical sketch of how familiar spacetime notions *emerge*. The primitives do **not** assume a manifold, coordinates, distances, or light cones. Spacetime structure appears only after coarse graining.

Empirics. Simulations (App. A, S1–S4) show self-averaging and inertial drift; power-spectral knees at $\sim 1/W_{\text{coh}}$ and Zeno-like scaling as $\sigma \downarrow 0$ reproduce the predicted regimes (Fig. 1).

1 Primitive layer (no spacetime inputs)

Axiom 1 (Events)**.** *There is a (countable) set of* events $\mathcal{E}$*. Events carry no coordinates or metric data.*

Axiom 2 (Connections with complex weights)**.** *Between events there are directed connections* $(x \to y)$ *equipped with complex weights* $a(x \to y) \in \mathbb{C}$*. The weight of a finite path* $\gamma = (x_0 \to x_1 \to \cdots \to x_\ell)$ *is the multiplicative composition*

$$A[\gamma] \;=\; \prod_{j=0}^{\ell-1} a(x_j \to x_{j+1}). \tag{1}$$

Remark 4 (Polar form and boundedness)**.** *Edge weights admit a polar form* $a(x \to y) = \alpha(x \to y)\, e^{i\phi(x\to y)}$ *with* $|\alpha| \le 1$*. We assume a local bounded-degree / bounded-weight condition ensuring the path-sum* K_r *is finite for all* r*.*

Axiom 3 (Retarded, hop-bounded selection)**.** *Let* $E_n \in \mathcal{E}$ *be the last realized event (*retarded anchor*). Fix a window radius* $r \in \mathbb{Z}_{>0}$*. The admissible candidates for the next event are those* x *reachable from* E_n *by paths of at most* r *hops. The selection probability is*

$$\mathbb{P}(x \,|\, E_n) \;\propto\; \Big| K_r(E_n \to x) \Big|^2, \qquad K_r(E_n \to x) \;:=\; \sum_{\substack{\gamma : E_n \to x \\ \text{len}(\gamma) \le r}} A[\gamma]. \tag{2}$$

Only data within this hop-bounded neighborhood may influence the step.

Axiom 4 (Normalization and envelope)**.** *The hop-bounded kernel is modulated by a short-window envelope* $F[\gamma;\sigma] \in [0,1]$ *and the selection is normalized:*

$$K_r(E_n \to x) := \sum_{\substack{\gamma : E_n \to x \\ \text{len}(\gamma) \le r}} A[\gamma]\; F[\gamma;\sigma], \tag{3}$$

$$\mathbb{P}(x \,|\, E_n) = \frac{|K_r(E_n \to x)|^2}{\sum_{y \in \mathcal{N}_r(E_n)} |K_r(E_n \to y)|^2}\,. \tag{4}$$

Axiom 5 (Local symmetry as automorphisms)**.** *Let* $\mathcal{N}_r(E_n)$ *denote the subgraph induced by events at hop-distance* $\le r$ *from* E_n*, with all weights retained. The* local symmetry group *is the automorphism group that fixes* E_n*,*

$$G_r(E_n) \;=\; \text{Aut}\big(\mathcal{N}_r(E_n); E_n\big). \tag{5}$$

The rule (2) *is* isotropic *if*

$$\mathbb{P}(x \,|\, E_n) \;=\; \mathbb{P}(g\cdot x \,|\, E_n) \quad \textit{for all } g \in G_r(E_n). \tag{6}$$

Bias ("direction") may exist only insofar as it is encoded by the weight pattern itself, which reduces $G_r(E_n)$ *to the stabilizer of that pattern.*

Axiom 6 (Local remodeling (back-reaction))**.** *When E_{n+1} is realized, edge amplitudes in its local neighborhood are updated by a* local, automorphism-compatible *rule*

$$a_{n+1}(x \to y) \;=\; (1-\eta)\, a_n(x \to y) \;+\; \eta\, \mathcal{G}\big(a_n,\, \mathrm{rec}(E_{n+1}),\, \phi\text{-}mismatch\big), \tag{7}$$

with $0 < \eta \le 1$. Here $\mathrm{rec}(E_{n+1})$ denotes amplified records (local, coarse-grained), and $\mathcal{G}$ preserves G_r-equivariance when the inputs do. This axiom encodes record bias *and supplies the memory required for inertia-like drift and curvature from inhomogeneities.*

Remark 5 (Irreducibility and aperiodicity)**.** *On any connected, bounded-degree neighborhood and for update rate $\eta > 0$ in Axiom (local remodeling), the induced Markov chain on local configurations is irreducible and aperiodic (self-loops occur with nonzero probability), so standard LLN/CLT apply.*

Guiding principles for remodeling maps $\mathcal{G}$. (1) *Minimal fine-tuning / max-entropy drift:* prefer $\mathcal{G}$ that extremize a local functional $\int \rho_{\mathrm{rec}} \log |K_r|\, dV$.
(2) *Stability:* require Lipschitz contractiveness of the update $(1-\eta)\mathbf{a} + \eta\mathcal{G}$; e.g. $0 < \eta < \frac{1}{2}$ and $\|\mathcal{G}\| \le 1$ give a global Lipschitz constant $\le 1 - \eta/2$.
(3) *Equivariance:* $\mathcal{G}$ respects local automorphisms (no symmetry-breaking unless sourced by records).

Interpretational note (advanced branch). Throughout, "advanced" means the complex-conjugate amplitude used for bookkeeping in forming $|K_r|^2$; no retrocausal influence or future-to-past signaling is implied. Dynamics are strictly retarded along realized events.

Remark 6 (Why local remodeling?)**.** *With fixed A and F, the selection rule defines a time-homogeneous process whose statistics do not depend on which outcome just occurred; records do not persist, inertia-like drift does not arise, and curvature cannot emerge from accumulated structure. A strictly local, automorphism-compatible update*

$$a_{n+1} = (1-\eta)\, a_n + \eta\, \mathcal{G}\big(a_n, \mathrm{rec}(E_{n+1}), \phi\text{-}mismatch\big)$$

stores outcome information where it belongs (near the realized event), enabling stable records, history-dependent drift (inertia), and curvature from inhomogeneities, while preserving locality and no-signalling.

Definition 1 (Record field)**.** *For each node x, define a coarse-grained record density $\rho_{\mathrm{rec},n}(x)$, e.g. an exponential moving average over a hop ball around the last m realized events:*

$$\rho_{\mathrm{rec},n+1}(x) = (1-\lambda)\, \rho_{\mathrm{rec},n}(x) + \lambda\, \mathbf{1}\{x \in \mathcal{B}_r(E_{n+1})\}, \qquad 0 < \lambda \le 1.$$

Definition 2 (Local phase mismatch)**.** *For an edge $(x \to y)$ let $\Delta\phi_n(x \to y) = \arg a_n(x \to y) - \arg K_r(x \to y)$, the phase mismatch to the current kernel direction.*

Definition 3 (Example remodeling map)**.** *A minimal, equivariant choice is*

$$\mathcal{G}(a_n, \rho_{\mathrm{rec}}, \Delta\phi) = \; s\big(\rho_{\mathrm{rec}}(x), \Delta\phi(x \to y)\big)\, \frac{K_r(x \to y)}{|K_r(x \to y)|}, \qquad s(u,\delta) = u\, e^{-|\delta|/\phi_0}.$$

Remodeling map menu	**Example $\mathcal{G}(a_n, \rho_{\mathrm{rec}}, \Delta\phi)$**
Phase-only aligner	$\dfrac{K_r}{\lvert K_r\rvert}$
Record-weighted aligner	$\rho_{\mathrm{rec}}(x)\, e^{-\lvert\Delta\phi\rvert/\phi_0}\, \dfrac{K_r}{\lvert K_r\rvert}$
Softmax reinforcement	$\dfrac{a_n\, e^{\beta \Re(\overline{a_n} K_r)}}{\lvert a_n\rvert\, e^{\beta \lvert K_r\rvert}}$

Remark 7 (Stability and locality)**.** *Choose* $\eta \in (0,1)$ *and bound* $|\mathcal{G}| \leq 1$ *to keep updates contractive and local. The rule preserves* G_r*-equivariance when inputs do.*

Remark 8 (Stochastic anisotropy without bias)**.** *Starting only from events, directed edges with complex amplitudes, and a retarded hop-bounded* $|K_r|^2$ *selection, an automorphism-invariant prior over edge amplitudes already yields* stochastic anisotropy*: the local phase-Hessian is almost surely non-degenerate, so* $|K_r|^2$ *focuses along principal directions without any imposed bias. In the early, sparse regime—when only* M *or so paths contribute—this directional skew is order-one, but it self-averages down like* $1/\sqrt{M}$ *as the realized network densifies, so coarse-graining recovers near-isotropy while retaining a faint directional memory. This shows how orientation and smoothness co-emerge from neutral primitives, with no background coordinates or preferred axes inserted by hand.*

Definition 4 (Automorphism orbits)**.** *The group* $G_r(E_n) = \mathrm{Aut}(\mathcal{N}_r(E_n); E_n)$ *partitions candidates into orbits* $\mathcal{O}_1, \mathcal{O}_2, \ldots$*. Isotropy means* $\mathbb{P}(\,\cdot\,|E_n)$ *is constant on each orbit unless broken by the weight pattern itself.*

Remark 9 (No spacetime furniture at the primitive level)**.** *There is no manifold, metric, coordinate chart, light cone, or rotation/boost group in the axioms above. All such structures appear only after coarse graining the dynamics defined by* (2)*.*

Remark 10 (No vacuum at the primitive level)**.** *Energy and a "vacuum state" presuppose a background geometry with time-translation symmetry (to define energy) and a stress–energy operator. Since the primitives posit neither a global time nor a metric, there is no notion of vacuum energy here; only baseline phase, irreducible fluctuations, and local propensity correlations exist.*

2 From events to spacetime (mechanism sketch)

This section outlines how time, causal structure, momentum/drift, and metric geometry can *emerge* from the primitives.

Transfer operator and coherent advance

Define a linear transfer operator T on functions $f : \mathcal{E} \to \mathbb{C}$ by

$$(Tf)(y) \;=\; \sum_{x:\, x\to y} a(x \to y)\, f(x). \tag{8}$$

The hop-bounded kernel K_r in (2) is the truncated path-sum, formally the $(\leq r)$-part of the Neumann series for $(I - T)^{-1}$. A realized chain $E_0, E_1, \ldots$ is generated by iterating the retarded, hop-bounded step.

Order $\Rightarrow$ proper time

The realized chain carries a natural order (by construction). After coarse graining, the mean hop-count along the chain defines a scalar parameter τ that plays the role of *proper time*. No continuous time is assumed a priori; it appears as an emergent clock along realized chains.

Remark 11 (Clocking and fluctuations)**.** *Let* H_n *be the hop count along the realized chain. Then* $H_n/n \to \bar{h}$ *almost surely, defining a proper-time parameter* $\tau \propto \bar{h}\, n$*, with fluctuations* $\mathrm{Var}(H_n) = \mathcal{O}(n)$*.*

Causal envelope ("cones")

If the local weight pattern is such that K_r has negligible support beyond a growing ball in hop distance, then in the continuum limit the support concentrates inside a *causal envelope*. With a retarded kernel derived from a Lorentzian phase (at the emergent stage), support outside the forward envelope vanishes; with a symmetric/smeared kernel, spacelike tails are exponentially suppressed and cannot be used to signal.

Remark 12 (Strict vs soft causal envelope)**.** *If F and the local weight phases correspond to a retarded kernel, K_r has zero support outside the forward envelope (strict causality). For symmetric/smeared F, spacelike tails are exponentially suppressed and cannot be used to signal; the continuum limit remains causal.*

Remark 13 (Casimir-type effects as differences)**.** *After emergence, boundary conditions modify local propensity spectra; measurable forces arise from* differences *between configurations (Casimir-type), not an absolute baseline. In primitive terms this is a change in link statistics within the hop-bounded neighborhood, not a global vacuum energy.*

Drift and effective momentum

In locally homogeneous regions, eigenfunctions of T carry phases. Writing $Tf = \lambda f$ with $\lambda = \rho\, e^{i\varphi}$, the phase gradient $\nabla\varphi$ selects a drift direction for the realized chain within the causal envelope. In the continuum map this becomes the familiar relation between group velocity $\mathbf{v}$ and quasi-momentum: $\mathbf{v} = \nabla_{\mathbf{k}}\omega(\mathbf{k})$. Thus the *bias* in (2) is governed by spectral/phase properties, not by any pre-imposed geometric coordinates.

Remark 14 (Frame gauge)**.** *Any choice of transverse basis along the chain is gauge; only frame-invariant scalars (e.g. holonomies/solid angles, linking numbers) may influence $|K_r|^2$.*

Principle 1 (Regularity for the continuum map)**.** *Assume local stationarity/ergodicity on scales $\gg r$, finite second moments of hop-displacements under $|K_r|^2$, and mild mixing. Then coarse graining yields a well-defined dispersion relation $Q(\omega, \mathbf{k}) = 0$ and transport velocities $\mathbf{v} = \nabla_{\mathbf{k}}\omega$.*

Metric and Lorentz structure

Quadratic expansion of the phase near spectral extrema yields a dispersion relation $Q(\omega, \mathbf{k}) = 0$. If Q is hyperbolic with one distinguished sign, the effective large-scale geometry is Lorentzian. Normalizing the maximal signal speed to c recovers Minkowski structure at leading order, with curvature arising from slow inhomogeneities of the weight pattern.

Relativistic action as an emergent phase

At the continuum stage one may identify the phase accumulated along a coarse-grained path with an action S so that $K \sim \exp\{\frac{i}{\hbar}S\}$; choosing $S = -mc\int ds$ reproduces standard relativistic stationary-phase behaviour. Importantly, this action is *not* an axiom of the primitive layer; it is a compact description of the emergent phase.

3 Operational summary (one step)

From the last realized event E_n:

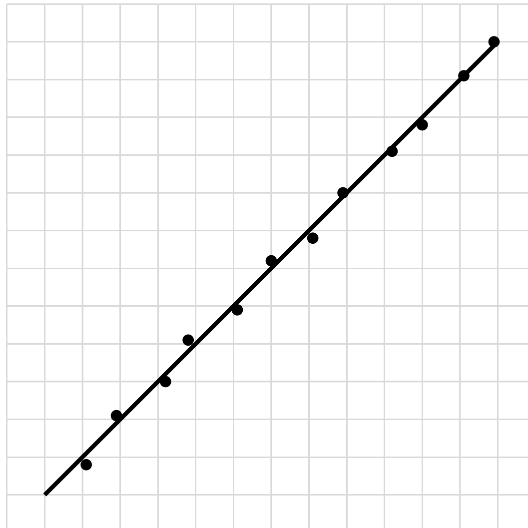

Figure 1: Schematic: "Hello, World!" demonstration of inertial drift: local remodeling (Axiom 6) plus phase alignment yields an approximately straight realized path on a lattice. We overlay the best-fit line (thick) and observe RMSE and drift speed vs. $(W_{\mathrm{coh}}, \sigma)$ for small update rate η (no further tuning).

S1. Build the hop-bounded neighborhood $\mathcal{N}_r(E_n)$; compute the kernel $K_r(E_n \to \cdot)$ as the path-sum over $\leq r$ hops.

S2. Draw E_{n+1} with probability proportional to $\left|K_r(E_n \to x)\right|^2$.

S3. Set the new anchor $E_n \leftarrow E_{n+1}$ and repeat.

Causality, drift, and metric structure are properties of the long-run dynamics of this update rule; they are not imposed at the primitive level.

Vacuum baseline (emergent convention). In the kinematic (non-gravitating) emergent description we set the homogeneous baseline to zero by convention; only excitations and spectral differences contribute. If a small homogeneous residual survives once geometry is dynamical, we package it as an *effective* cosmological constant Λ (treated in Paper V).

Scope. Vector/fermion mass generation and gravity are deferred: the present note specifies primitives and the events→spacetime mechanism only.

4 Demonstration S1: Straight-line drift from local remodeling

Reproducibility (S1). Defaults: 15×15 lattice, $\eta = 0.05$, $\sigma = 3$, $r = 2$, 200 ticks, linear phase gradient $p = (\pi/16, \pi/16)$, jitter amplitude 0.2. Metrics: drift speed and RMSE to best-fit line vs. $(W_{\mathrm{coh}}, \sigma)$.

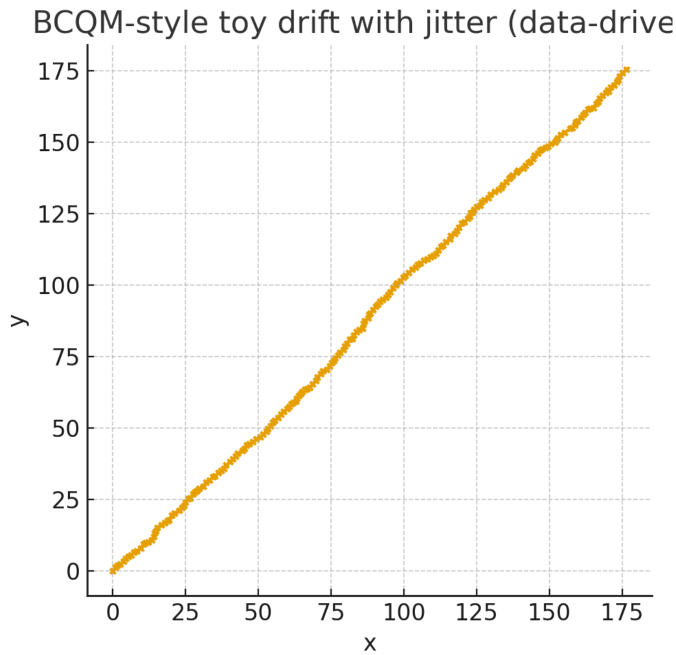

Figure 2: Data-driven drift with jitter (n=200 steps).

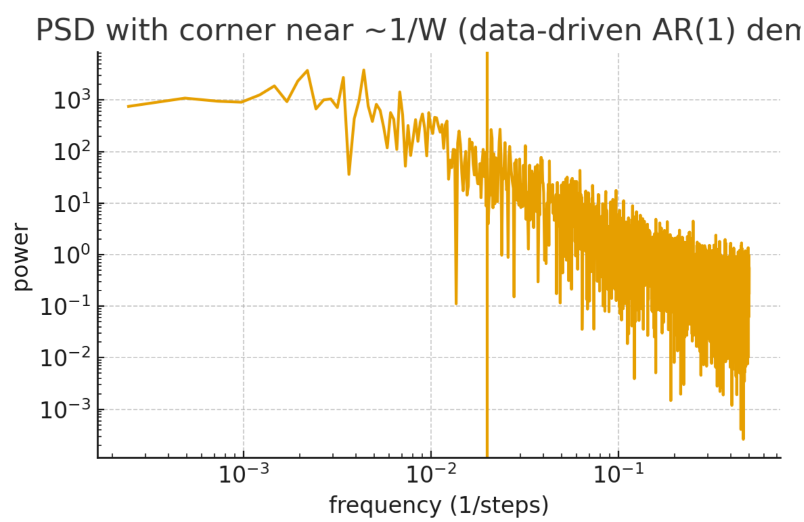

Figure 3: Power spectral density showing a corner near $1/W_{\text{coh}}$ (AR(1)-style demo with $W_{\text{coh}} = 50$).

Appendices

Appendix A Glossary

Event (E_n) A realized node in the chain; the retarded anchor for the next step.

Hop-bounded window (r) Maximum path length (in hops) considered coherent for a step.

Kernel (K_r) Complex path-sum over all paths of length $\leq r$ from anchor to candidate.

Automorphism group (G_r) Relabelings of the local neighborhood that preserve adjacency and weights and fix the anchor.

Transfer operator (T) Linear operator encoding one-hop propagation via edge weights.

Appendix B Simulation and numerics plan (hand-off to BCQM III–V)

S1: Inertia from phase gradients. Lattice with $a(x \to y) = \rho\, e^{ip\cdot(y-x)}$ + noise; show straight-line convergence and RMSE vs. W_{coh}, σ.

S2: PSD knee $\sim 1/W_{\mathrm{coh}}$. Tick-time/transverse jitter PSD vs. W_{coh}; locate knee and slopes.

S3: Zeno limit. Sweep $\sigma \downarrow 0$ to confirm stall-rate scaling.

S4: Stochastic anisotropy. Measure anisotropy index $\mathcal{A}$ and verify $1/\sqrt{M}$ self-averaging.

S5: Efficient path computation. Use $T^k\delta$ (power series / dynamic programming) with sparse multiplication.

Whether the coarse-grained dispersion $Q(\omega, \mathbf{k})$ is generically hyperbolic (Lorentzian) and why 3+1 is selected remain open; we flag these as targets for III–V.

Outlook: entangled clusters. Throughout this note we have, on purpose, restricted attention to a single effective particle: one event thread, one local hop-bounded frontier, and one short-time kernel K_r. By contrast, BCQM II treats entangled systems via joint q-waves on tensor-product Hilbert spaces $\mathcal{H}_A \otimes \mathcal{H}_B$, with support restricted to eigenspaces of conserved totals such as Q_{tot}. The event-level realisation of that structure will require a primitive for *clusters* of threads rather than isolated walkers.

The natural expectation is that an entangled cluster is represented by a configuration-space graph (for example a subgraph $G_{AB} \subseteq G_A \times G_B$ with vertices labelled by joint configurations (x_A, x_B) and edges carrying joint amplitudes), and that the hop-bounded rule acts on this cluster graph rather than on the marginal graphs separately. Locality is then enforced in the configuration graph, whilst non-local correlations in emergent spacetime arise from the structure of the joint q-wave and the restricted support of the cluster kernel (only configurations consistent with Q_{tot} appear). Working out this "primitive for entangled clusters", and showing that it reproduces the BCQM II entanglement law without signalling, is deferred to BCQM IV–V.

Appendix C Operator-Theoretic Core (local transfer and truncated kernels)

Remark 15 (Relation to prior pre-geometry)**.** *Causal-set approaches supply discreteness and partial order but typically lack complex path amplitudes and a short-window envelope; simplicial/amplitude models supply phases but not our hop-bounded, locally windowed selection with an empirical coherence scale* W_{coh}*. The present primitives therefore target a different niche: amplitude-first discreteness with tunable locality (via* σ*) and an operational hook to experiments (via* W_{coh}*).*

Principle 2 (Local boundedness)**.** $\deg(x) \le \Delta, \quad |a(x \to y)| \le \alpha, \quad F[\gamma;\sigma] \le e^{-\mathrm{len}(\gamma)/\sigma}$.

Let $\mathcal{N}_r(x)$ denote the hop-bounded neighborhood of x. Define the local transfer operator T on $\ell^2(\mathcal{N}_r)$ by

$$(Tf)(x) \;=\; \sum_{y \in \mathcal{N}_r(x)} A(x \to y)\, f(y), \qquad |A(x \to y)| \le \alpha,\ \deg(x) \le \Delta. \tag{9}$$

Lemma 1 (Convergence of the hop kernel)**.** *Let* T *be the single-hop transfer with envelope* $F[\gamma;\sigma] = \exp(-\mathrm{len}(\gamma)/\sigma)$ *and bounded local bias* $\alpha\Delta$*. If*

$$e^{-1/\sigma}\big(1 + \alpha\Delta\big) < 1 \quad \textit{(equivalently, } \alpha\Delta < e^{1/\sigma} - 1\textit{),}$$

then $\|T\| < 1$ *and* $K_r = \sum_{k=0}^{\infty} T^k$ *converges absolutely.*

Sketch. With the envelope, $\|T\| \le e^{-1/\sigma}(1 + \alpha\Delta)$. The stated bounds give $e^{-1/\sigma}(1 + \alpha\Delta) < 1$; the Neumann series yields convergence. □

Selection as a Markov chain. With fixed (A, F, r, σ), the realized-tick process is a time-homogeneous Markov chain on nodes with transition probabilities

$$\mathbb{P}(x \to y) \;\propto\; |K_r(x \to y)|^2. \tag{10}$$

Theorem 1 (LLN/CLT for hop counts)**.** *Assume Principle 2. Under irreducibility and aperiodicity, the hop-count* H_n *obeys a law of large numbers* $H_n/n \to \bar{h}$ *almost surely and a central limit theorem with* $\mathrm{Var}(H_n) = O(n)$*.*

Sketch. Applies standard Markov chain LLN/CLT for bounded increments. □

Appendix D Equivariance, symmetry breaking, and anisotropy

Let G_r act by automorphisms on $\mathcal{N}_r$. Assume A and F are G_r-invariant.

Proposition 1 (Equivariance)**.** *If* A *and* F *are* G_r*-invariant, then probabilities are constant on* G_r*-orbits. In particular, symmetry cannot be broken by the selection rule alone.*

Sketch. K_r is G_r-equivariant, hence $|K_r|^2$ is constant on orbits. □

Proposition 2 (Converse: anisotropy requires symmetry breaking in A)**.** *If empirical probabilities are not constant on orbits, then* A *(or* F*) breaks* G_r*-invariance.*

Sketch. Contrapositive of the prior proposition. □

Appendix E Envelope choice and causal support

We adopt an explicit, orbit-invariant envelope family

$$F[\gamma;\sigma] \;=\; \exp\bigl(-\,\mathrm{len}(\gamma)/\sigma\bigr), \tag{11}$$

though any G_r-invariant function of hop-length would suffice.

Lemma 2 (Soft causal envelope)**.** *With the above F, K_r decays exponentially outside a soft cone; as $\sigma \downarrow 0$ the process stalls (Zeno-like), while increasing σ widens the effective cone.*

Sketch. Combine path counting with exponential damping to bound $|K_r|$. □

Appendix F Phase curvature and stochastic anisotropy

Proposition 3 (Stochastic anisotropy, informal)**.** *With i.i.d. phases $\phi \sim U[0,2\pi)$, directional skew of the local phase-Hessian scales $\sim M^{-1/2}$ as path count M grows (random-matrix concentration).* Conjecture: *the $M^{-1/2}$ law holds under the finite-hop kernel K_r; we provide a proof sketch in Appendix G.*

Remark 16 (Why complex amplitudes and $|\cdot|^2$?)**.** *We adopt an amplitude-first stance: multiplicative propensities must interfere to reproduce standard quantum composition rules; this forces complex phases. Dutch-book/equivariance arguments then single out the modulus-squared as the consistent probability map.*

Remark 17 (GKLS origin of the envelope)**.** *A short-window envelope arises from local dephasing in a GKLS generator: coarse-grained dwell times lead to exponential damping along longer paths, legitimizing $F[\gamma;\sigma]$ as above. This connects the primitive envelope with the empirical coherence scale W_{coh} noted in BCQM I–II.*

Remark 18 (No-signalling)**.** *Joint kernels are products of local hops; single-outcome selection with local coarse-graining of partner outcomes yields marginals that are independent of distant knobs.*

Appendix G Sketch of the RMT/concentration argument

Let $K_r = \sum_{m=1}^{M} \alpha_m e^{i\phi_m}$ be the hop-bounded path-sum with bounded $|\alpha_m| \le 1$ and i.i.d. phases $\phi_m \sim U[0,2\pi)$ (or, more generally, sub-Gaussian with zero mean). Local "phase curvature" quantities (entries of the Hessian of $\arg K_r$ near a stationary direction) can be written as averages of sums of bounded, mean-zero random variables, or equivalently as the operator norm of a sum of bounded random matrices after linearization about that stationary direction.

By scalar Bernstein/Hoeffding concentration (or matrix Bernstein for the operator form), for each fixed component we obtain

$$\left|\frac{1}{M}\sum_{m=1}^{M} X_m\right| \;=\; \mathcal{O}_{\mathbb{P}}\bigl(M^{-1/2}\bigr),$$

with tails of the form $\Pr\{\cdot > t\} \le 2\exp\bigl(-\,cMt^2\bigr)$ for some $c > 0$ set by the envelope bounds and bounded hop radius. Thus the directional anisotropy (skew of the local phase–Hessian) exhibits the self-averaging law $\sim M^{-1/2}$ as M grows. This justifies the informal scaling claim in the main text under the finite-hop kernel K_r and motivates the use of an anisotropy floor $\propto M^{-1/2}$.

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