Boundary-Condition Quantum Mechanics (BCQM)

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Abstract

Boundary-Condition Quantum Mechanics (BCQM) extends standard quantum theory by retaining both advanced (t^-) and retarded (t^+) solutions of the time-dependent Schrödinger equation. This dual-time structure is constrained by a finite collapse horizon W, which sets the boundary between reversible superposition and irreversible collapse. W is empirically anchored in circuit-QED experiments such as Minev *et al.*, and can in principle be measured across multiple platforms. A speculative symmetry partner, a re-coherence horizon V, is noted but not required. We *apply* a quadratic realisation law $\mathbb{P} \propto |\psi_+|^2$, motivated by pairing the retarded (t^+) amplitude with its conjugate. A derivation under explicit axioms is provided in *Analytical Proofs for BCQM* [1]. Ensemble statistics of collapse ticks in our models agree with this weighting. The framework respects conservation laws, no-signalling, and bounds from Bell and Tsirelson, and aligns with Sorkin's hierarchy. Classic quantum puzzles, including entanglement correlations, delayed-choice behaviour, Zeno effects, and the arrow of time, receive consistent resolution within this boundary-condition picture. BCQM is therefore falsifiable: collapse horizons provide quantitative predictions, and the puzzle resolutions follow without altering the standard modulus–square weighting. Future work will focus on measuring W, probing rollback close to its limit, and testing whether re-coherence signatures associated with V can be experimentally realised. *Clarification.* Throughout this work, the "advanced (t^-) solution" refers to the advanced branch of the TDSE retained to respect time symmetry at the level of amplitudes. It is *not* interpreted as evolution "back in time," carries no causal influence from the future, and cannot be used for signalling; operational time-ordering remains on the ordinary chronological axis t^+.

1 Introduction

Quantum mechanics provides unmatched predictive power, yet its standard formulation leaves the mechanism of collapse unresolved. Decoherence theory explains how superpositions become locally classical, but it does not define when or why a single outcome is realised. The measurement problem therefore remains open.

Boundary-Condition Quantum Mechanics (BCQM) is proposed as a minimal extension to standard quantum mechanics that addresses this gap without altering its successful predictions. BCQM rests on two novelties:

1. Both retarded and advanced solutions of the time-dependent Schrödinger equation are taken on equal footing, forming a dual-axis probability structure (the q-wave).
2. Collapse is governed by a finite coherence horizon W, beyond which rollback of interference is impossible, and one p-wave path is stochastically realised. By symmetry a re-coherence horizon V is also proposed, with support from experimental recovery data, though it plays a more speculative role.

The time-dependent Schrödinger equation is time-symmetric and admits both retarded (t^+) and advanced (t^-) solutions. Conventional quantum mechanics discards the advanced branch, treating it as unphysical. BCQM retains both, not as a speculative addition but as the minimal choice consistent with the underlying symmetry. In this work we *apply* the modulus-square weighting for realised outcomes, motivated by pairing the retarded amplitude with its conjugate; the formal derivation is given in *Analytical Proofs for BCQM* [1].

Interpretation note (no retrocausality). In BCQM the advanced component t^- is retained purely as a mathematical completion of the time-symmetric TDSE. We use t^- as a *coherence axis* of the q-wave—not as a physical timeline running to the past. All causes, records, and interventions occur along t^+; the t^- component neither transmits information nor changes past events.

Relation to other approaches. BCQM differs in kind from existing alternatives to orthodox quantum mechanics. Spontaneous-collapse models such as GRW [2] introduce explicit stochastic terms; Penrose's objective reduction [3] appeals to gravitational instability; and pilot-wave theory [4] relies on hidden variables and a guiding wave. By contrast, BCQM retains the standard unitary dynamics but introduces finite boundary conditions on the q-wave. Collapse is thereby enforced through horizons rather than new dynamics or hidden parameters.

The following sections set out the framework. Section 2 introduces the glossary of terms and postulates. Section 6 presents the mathematical formulation. Section 7 illustrates the framework with a computational model. Section 8 describes collapse dynamics. Consistency checks and empirical implications are addressed in later sections.

Scope and novelty. BCQM is not a hidden-variable theory and introduces no new stochastic dynamics. It reproduces standard quantum predictions in regimes where the coherence horizon W is large, but posits a finite, testable boundary condition that sets when rollback of superpositions becomes impossible. In this sense BCQM is a falsifiable *extension* of orthodox QM rather than a mere reinterpretation. The specific quantum puzzles addressed by the framework are outlined in Sec. 4.

The complete manuscript is archived on Zenodo and can be cited as [5].

2 Definitions and Framework

2.1 Particle

The particle is the single, localised carrier of conserved quantities such as energy, momentum, and charge. It is not distributed across the q-wave. The particle is realised at emission and at collapse, with intermediate travel treated as a discontinuous jump guided by the probability structure of the q-wave.

2.2 Q-Wave

The q-wave is the dual-axis probability structure formed from both the retarded and advanced solutions of the time-dependent Schrödinger equation. It is inseparable from the particle's existence but distinct in nature: an abstracted property that is not physical and carries no conserved quantities. The q-wave is decomposed by Fourier analysis into dominant modes, which shape the probability paths (p-waves) representing the particle's probable outcomes.

2.3 P-Wave Paths

p-waves are the probability paths derived from the Fourier decomposition of the q-wave. They represent probable outcomes but are not physical trajectories. Collapse corresponds to the stochastic realisation of one p-wave path, weighted by the q-wave amplitudes.

2.4 Collapse Horizons

Collapse is bounded by two horizons. The coherence horizon W sets the maximum extent of recoverable interference: beyond W, rollback is impossible, and collapse is inevitable. By symmetry, a re-coherence horizon V is proposed to describe the up-ramp of recovery; V is supported by experimental data but remains speculative.

2.5 Collapse

Collapse is reached once the coherence horizon W has been exceeded. At this point rollback is no longer possible, and one p-wave path is stochastically realised, weighted by the q-wave amplitudes. The q-wave itself does not collapse; it continues to encode all possible p-wave paths. Collapse is irreversible but remains consistent with conservation laws and no-signalling.

2.6 Glossary of key terms

For clarity, we summarise the central terms introduced in BCQM:

- **q-wave:** abstract probability structure derived from both advanced (t^-) and retarded (t^+) solutions of the TDSE; not a physical wave.
- **p-wave:** decomposed possibility paths within the q-wave, encoding all allowed outcomes.
- **Particle:** the entity that carries conserved quantities and appears discontinuously at collapse; guided by the q-wave but not contained in it.
- **Collapse horizon W:** finite boundary beyond which collapse becomes irreversible; empirically anchored in circuit-QED experiments.
- **Re-coherence horizon V:** speculative symmetric partner to W, permitting possible re-emergence of coherence; not yet observed.
- **Collapse:** stochastic realisation of one p-wave path once W is exceeded; ensemble behaviour reproduces the modulus-square (Born) weighting.
- **advanced (t^-) solution:** the advanced TDSE branch retained for time symmetry of amplitudes; used as a coherence-axis label, not a "backwards-in-time" evolution or signalling resource.

3 Postulates and Ontology

3.1 Dual TDSE Solutions

Both retarded (t^+) and advanced (t^-) solutions of the Schrödinger equation are retained, reflecting its intrinsic time symmetry. In this work we *apply* the quadratic weighting $w = |K|^2$; the pairing of t^+ with its conjugate motivates this form, and a formal derivation is given in *Analytical Proofs for BCQM* [1]. For clarity, here K denotes the retarded local kernel $K_r^{(+)}(x \mid E_n)$ at the candidate event, cf. Appendix G, Eqs. (26)–(27).

Retention of both time directions. Standard quantum mechanics discards the advanced solution of the time-dependent Schrödinger equation as non-physical. In BCQM, by contrast, both retarded (t^+) and advanced (t^-) components are retained. This is not an additional postulate but the minimal completion consistent with the time symmetry of the underlying equation. The t^- contribution does not introduce new dynamics; it extends the probability structure so that interference and collapse can be described with respect to both horizons. In this sense, t^- is the natural partner of t^+, required to preserve the full symmetry of the governing equation.

Clarification (advanced branch). Including $\psi^{(-)}$ leaves causal structure unchanged. Both $\psi^{(+)}(t)$ and $\psi^{(-)}(t)$ are functions of the same laboratory time t; the labels (t^+, t^-) are bookkeeping for the two amplitude branches. No protocol can use $\psi^{(-)}$ to signal to earlier times or to condition present outcomes on future choices.

3.2 Particle–Q-Wave Separation

The particle carries energy, momentum, charge, and other conserved quantities. The q-wave carries none of these; it encodes only the probability structure for outcomes.

3.3 Collapse Horizon W

Collapse is governed by a finite coherence horizon W. Before W, interference may be restored if records are erased. Once W is exceeded, rollback is impossible and collapse to a single outcome is inevitable.

3.4 Stochastic Realisation

Collapse corresponds to the point beyond which rollback is impossible, at which one p-wave path is stochastically realised, weighted by the q-wave amplitudes. The realisation is not determined by the most weighted path.

3.5 Conservation and Locality

Collapse does not violate conservation laws: the particle alone carries conserved quantities. Collapse is local and preserves no-signalling, Tsirelson bounds [6], and Bell consistency [7]. For composite systems, this statement applies only to the joint conserved total. The partition across subsystems is undefined until collapse, where particles realise their shares in a way consistent with the joint q-wave structure. **Clarification.** In BCQM the particle always carries conserved quantities, but for composite (entangled) systems these quantities are constrained only at the joint level. The total (e.g. total spin, energy, or charge) is fixed by symmetry, while the division across subsystems is left indeterminate until collapse. Collapse at the horizon W then stochastically realises a partition consistent with the joint q-wave structure. This ensures that entangled correlations respect conservation laws without requiring pre-assigned values to each subsystem.

3.6 Joint q-wave for composites

For composite systems, the q-wave is defined on the tensor product Hilbert space and encodes *joint* possibility paths. Conserved quantities appear as symmetries of the joint algebra: if $Q = Q_A + Q_B$ and $[H, Q] = 0$, then the total Q is invariant under all evolution. The partition across subsystems is not fixed until collapse or re-entry; at that boundary, particles realise their shares stochastically in a way consistent with the joint q-wave structure.

3.7 Relation to other interpretations

It is useful to note how BCQM differs from existing proposals. Unlike GRW or continuous-spontaneous-localisation models, BCQM does not add stochastic noise terms to the dynamics; instead, collapse is governed by the finite horizon W. Unlike Bohmian mechanics, BCQM does not posit a guiding wave with ontological status: the q-wave is an abstract probability structure only, with no physical substance. And unlike retrocausal models, the advanced component t^- is retained mathematically to preserve time symmetry but cannot be used for signalling or influence from the future. These contrasts highlight BCQM as a distinct framework rather than a relabelling of existing interpretations.

3.8 Ontology Note

The q-wave is not a physical wave but a probability structure. The particle is always in spacetime and realised at emission and collapse. Between these events, its travel is discontinuous, guided only by the q-wave probabilities.

4 Quantum puzzles addressed by BCQM

Standard quantum mechanics has long been associated with conceptual puzzles and apparent paradoxes. Boundary-Condition Quantum Mechanics (BCQM) does not introduce new dynamics but reframes collapse as a boundary-condition effect on the q-wave. Within this framework, several canonical puzzles are naturally resolved or reinterpreted.

4.1 Measurement problem

The timing and mechanism of collapse are ambiguous in orthodox formulations. In BCQM, collapse occurs once the coherence horizon W is exceeded: rollback of interference is no longer possible, and one p-wave path is stochastically realised. The q-wave itself remains intact; only the reversibility of p-wave path superpositions is lost.

4.2 Entanglement and non-locality

Quantum correlations appear to act instantaneously at a distance. In BCQM, both retarded (t^+) and advanced (t^-) solutions are equally retained in the q-wave. Correlations are set by shared boundary conditions, while locality and no-signalling remain preserved. *Clarification.* Here "shared boundary conditions" means the jointly prepared state and the local measurement settings at the time of each measurement; outcomes do not depend on settings chosen in the future, and no retrocausal signalling is implied. Bell-type inequalities are therefore still bounded by the Tsirelson limit

$$|S| \leq 2\sqrt{2}, \tag{1}$$

consistent with experiment.

BCQM resolution. Nonlocal correlations arise from joint-channel realisation at W within a conserved-Q sector of the joint q-wave. Local CPTP boundaries guarantee no-signalling, while the joint structure respects Bell and Tsirelson limits exactly as in standard QM. In this picture, total conservation is enforced at the joint level, while subsystem values are only specified at collapse. This prevents any hidden-variable assignment while ensuring that all realised outcomes remain consistent with the conserved total.

4.3 Delayed-choice and quantum eraser

Experiments where choices appear to retroactively determine interference challenge classical causality. In BCQM, rollback of p-wave path superpositions is possible until the horizon W is reached. After W, outcomes are stochastically realised, consistent with quantum eraser results and the mid-flight reversal of quantum jumps in circuit-QED [8].

No "back in time" reading. Apparent retroactive effects are explained here by boundary conditions and rollback before the horizon W, not by influences propagating from the future; BCQM is not a retrocausal theory.

4.4 Arrow of time and irreversibility

Microscopic laws are reversible, yet macroscopic behaviour is not. BCQM introduces irreversibility through the passing of the horizon W: once exceeded, collapse cannot be undone. The resulting evolution is described by unital CP maps Φ, which satisfy

$$S(\Phi[\rho]) \geq S(\rho), \tag{2}$$

so entropy is non-decreasing, consistent with Loschmidt's paradox and Poincaré recurrence.

4.5 Quantum Zeno effect

Repeated measurement can appear to freeze evolution. In standard QM, the survival probability after N projections of a state rotated by angle θ is

$$\left[\cos^2\left(\frac{\theta}{N}\right)\right]^N \xrightarrow[N\to\infty]{} 1, \tag{3}$$

producing the Zeno effect. The survival-probability's quadratic form matches the standard result; here we *apply* the modulus-square weighting motivated by pairing, with the derivation provided in *Analytical Proofs for BCQM* [1]. Collapse at the horizon W realises one outcome per run, and repeated runs expose this fixed weighting in the observed frequencies.

4.6 Contextuality

The Kochen–Specker theorem shows that measurement outcomes depend on context. In BCQM, the context is encoded directly in the q-wave decomposition, ... so p-wave path realisation is consistent with contextuality without contradiction.

4.7 Born rule

The origin of the squared-amplitude weighting is unresolved in standard QM. Probabilities are given by

$$P_i = |\langle i|\psi\rangle|^2$$

Remark (Provenance of the quadratic rule)**.** *The survival-probability's quadratic form matches the standard result; here we* apply *the modulus-square weighting motivated by pairing the retarded amplitude with its conjugate, with the derivation provided in* Analytical Proofs for BCQM *[1].*

Note on speculative extensions. A symmetric recovery horizon V has been conjectured by analogy with W, corresponding to a regime where re-coherence would become inevitable. This idea is included only as a speculative symmetry and is not part of the core BCQM proposal. All results presented in this paper rely solely on the existence of the collapse horizon W.

5 The Q-Wave Framework

The q-wave is the central construct of BCQM. It is formed by taking both the retarded and advanced solutions of the time-dependent Schrödinger equation on equal footing. This defines a dual-axis probability structure: the chronological axis t^+ along which the particle propagates, and the coherence axis t^- which sets the extent of recoverable interference. This t^- label denotes the advanced amplitude branch used for time symmetry—*not* a literal reverse-time evolution or any channel for retrocausal influence.

To expose usable structure, the q-wave is decomposed by Fourier analysis into dominant modes. These modes shape the probability paths (p-waves), which represent the lattice of probable outcomes available to the particle. The q-wave is inseparable from the particle's existence but distinct in nature: it is an abstracted property, not a physical wave. It carries no conserved quantities but encodes only the statistical scaffolding that guides the particle.

In BCQM, collapse occurs once the coherence horizon W has been exceeded, when rollback is no longer possible and one p-wave path is stochastically realised. The role of V as a re-coherence horizon is conjectured by symmetry and supported by experimental recovery dynamics, but plays only a secondary role. The q-wave framework therefore supplies the probability structure within which the particle's discontinuous emission–collapse dynamics are realised.

6 Mathematical Framework

6.1 Time-dependent Schrödinger equation

The standard evolution of a quantum system is given by

$$i\hbar \frac{\partial}{\partial t}\psi(t) = H\psi(t), \tag{4}$$

with Hamiltonian H comprising kinetic and potential contributions. Both the retarded and advanced solutions of Eq. (4) are retained in BCQM, combined to form the q-wave[1]:

$$q(t) = \psi^{(+)}(t) + \psi^{(-)}(t). \tag{5}$$

6.2 Fourier decomposition and p-wave paths

To expose structure, the q-wave is Fourier-decomposed into dominant modes. These modes define the probability paths (p-waves), representing the set of probable outcomes accessible to the particle. Collapse corresponds to the stochastic realisation of one p-wave path, weighted by q-wave amplitudes. In BCQM these amplitudes are not probabilities. Collapse realises a single p-wave path per run, and repeated runs *recover the applied* quadratic $|a_j|^2$ weighting; see *Analytical Proofs for BCQM* [1] for the derivation under explicit axioms.

6.3 Open-system dynamics

Environmental influence is treated with the Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) equation. For a system density matrix $\rho(t)$,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_j \left(L_j \rho L_j^\dagger - \tfrac{1}{2}\{L_j^\dagger L_j, \rho\} \right), \tag{6}$$

where L_j are Lindblad operators representing dissipative processes (standard quantum channels).

6.4 Entanglement, conservation, and locality

Consider a bipartite entangled state

$$|\Psi\rangle = \sum_{ij} c_{ij}\, |i\rangle_A \otimes |j\rangle_B, \qquad \rho = |\Psi\rangle\langle\Psi|.$$

Let a conserved quantity be $Q = Q_A + Q_B$ with $[H, Q] = 0$. Then $\langle Q\rangle_t = \mathrm{Tr}(Q\,\rho_t)$ is constant for all t.

[1] "Advanced" here refers to the solution obtained with the advanced Green's function. Both branches evolve in the same time parameter t; t^- does not denote negative time, only the advanced amplitude branch retained for time symmetry.

BCQM's boundary maps at W and (speculatively) V act *locally* on subsystems and are completely positive and trace preserving (CPTP). For example,

$$\rho \mapsto (\Phi_W^A \otimes \mathbb{I}_B)(\rho), \quad \rho \mapsto (\mathcal{P}_V^A \otimes \mathbb{I}_B)(\rho),$$

with idempotency $\Phi_W^2 = \Phi_W$ and $(\mathcal{P}_V)^2 = \mathcal{P}_V$.

Local CPTP action guarantees no-signalling:

$$\mathrm{Tr}_A\big[(\Phi_W^A \otimes \mathbb{I}_B)(\rho)\big] = \mathrm{Tr}_A(\rho) = \rho_B,$$

and likewise for $\mathcal{P}_V$. Thus local collapse or re-entry on A cannot alter B's reduced state without classical communication, while joint measurements still exhibit nonlocal correlations. Because $[H, Q] = 0$ and the maps commute with the Q-sector decomposition, total Q is conserved across the boundary; only the *partition* between subsystems is fixed stochastically when channels are realised.

6.5 Worked example: Ramsey-type qubit

For a two-level system with amplitude damping (rate Γ) and pure dephasing (rate γ_ϕ), the GKLS equation (6) specialises to

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \tfrac{\Gamma}{2}\mathcal{D}[\sigma_-]\rho + \gamma_\phi \mathcal{D}[\sigma_z]\rho, \tag{7}$$

with $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$. The off-diagonal coherence decays as

$$L(t) = \frac{|\rho_{01}(t)|}{|\rho_{01}(0)|} = e^{-(\Gamma/2+\gamma_\phi)t}. \tag{8}$$

Defining the coherence horizon W by the threshold condition $L(W) = \varepsilon$,

$$W = \frac{\ln(1/\varepsilon)}{\Gamma/2 + \gamma_\phi}. \tag{9}$$

For the conventional choice $\varepsilon = e^{-1}$, W reduces to the standard coherence time $T_2^* = 1/(\Gamma/2+\gamma_\phi)$.

For typical circuit-QED parameters [8], this yields $W \sim 4\,\mu$s; the re-coherence horizon V inferred from recovery dynamics is shorter, ~ 0.3–$0.5\,\mu$s.

6.6 Collapse horizon W

The coherence horizon W is defined by the smallest nonzero dissipative eigenvalue of the GKLS generator. In practice, this gives a finite timescale for which rollback of coherence is possible. Once $t > W$, rollback is impossible, and collapse is inevitable. W is therefore the measurable anchor of BCQM, with timescales consistent with circuit-QED data (e.g. [8]). The explicit connection between GKLS decay rates and the coherence horizon W is worked out in a toy model (Appendix B.1).

6.7 Re-coherence horizon V

By symmetry a re-coherence horizon V can be defined, describing the up-ramp of recoverable interference. Unlike W, V remains speculative, though supported by experimental recovery dynamics. It provides a conceptual mirror to W but is not formalised as a postulate.

7 Program Model

To illustrate the BCQM framework, a computational model was developed based on a Galton-style lattice. In this analogy the branches represent the p-wave paths derived from Fourier decomposition of the q-wave. The particle's collapse is simulated as a stochastic realisation among these p-wave paths, weighted by the q-wave amplitudes. The model does not track a continuous trajectory of the particle but implements discontinuous landings consistent with BCQM postulates.

7.1 Lattice analogy

Figure 1 shows the Galton-style lattice used to visualise the path structure. Each branch corresponds to a p-wave path, and the lattice as a whole represents the full set of probable outcomes available to the particle.

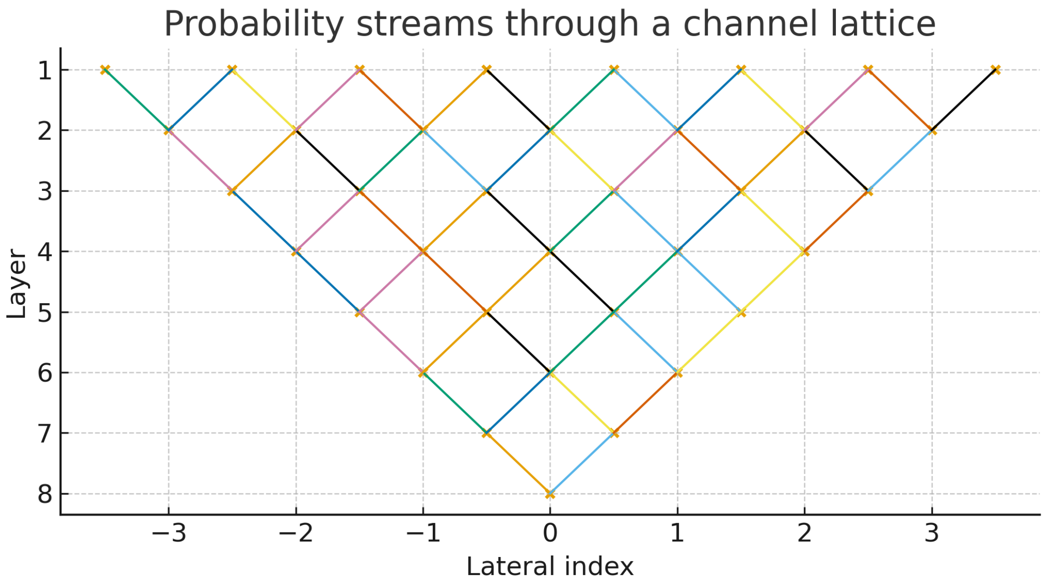

Figure 1: Galton-style path lattice used to represent the p-wave decomposition. Branch amplitudes are evolved under the TDSE and GKLS dynamics, and the particle's stochastic landing is drawn from the resulting weights.

7.2 P-wave path weights

The relative amplitudes of the p-wave paths evolve according to the q-wave. Figure 2 shows an example distribution of p-wave path weights, which set the probability of collapse into each outcome.

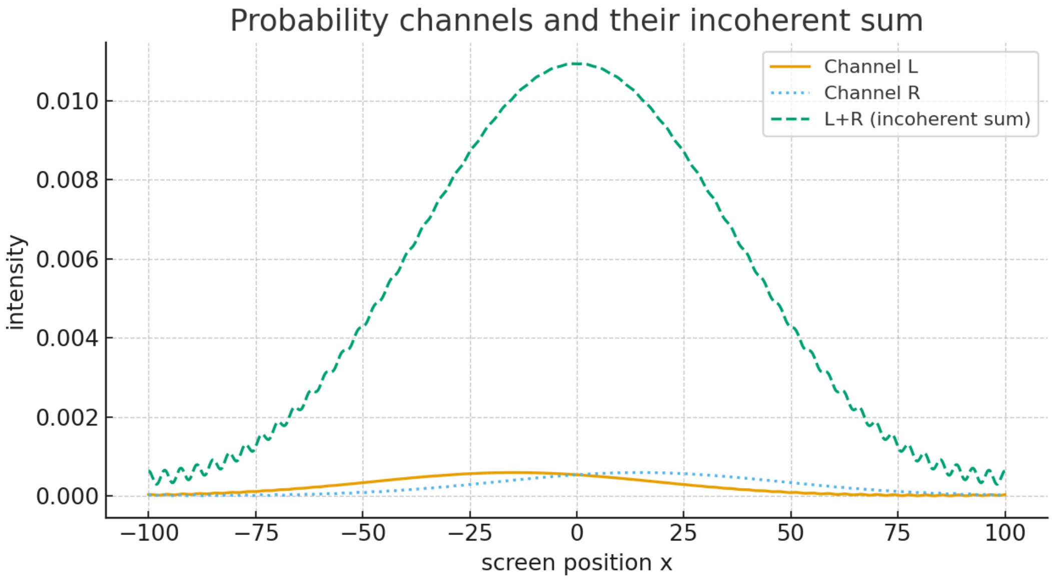

Figure 2: Distribution of p-wave path weights across the lattice leaves.

7.3 Collapse horizon

The program implements the coherence horizon W as a cutoff on recoverable interference. Figure 3 shows the coherence function $L(t)$ with collapse enforced once $t > W$. A schematic of both W and the proposed re-coherence horizon V is shown in Fig. 4.

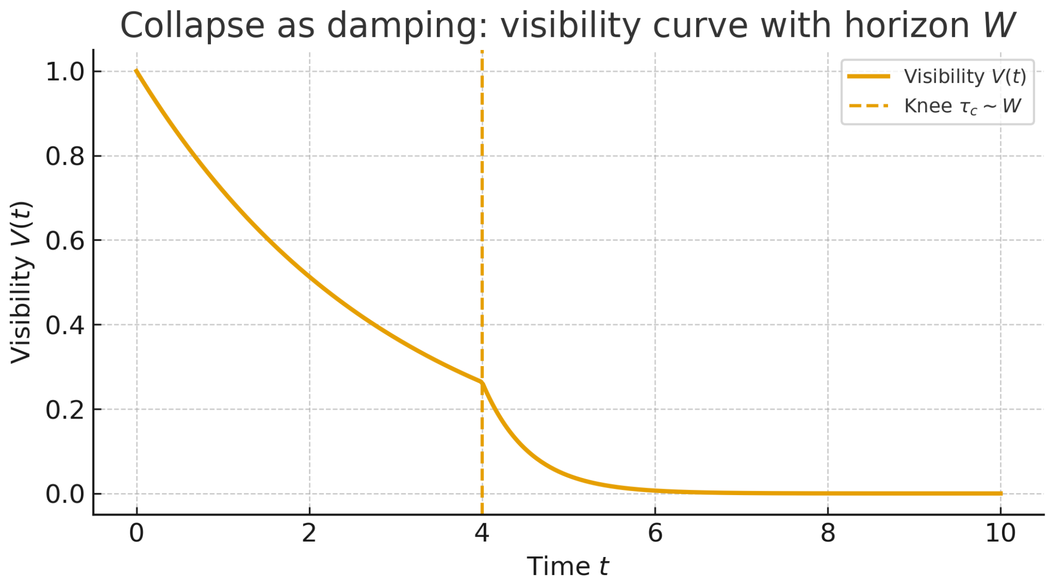

Figure 3: Collapse implemented by stochastic realisation once the coherence horizon W has been passed.

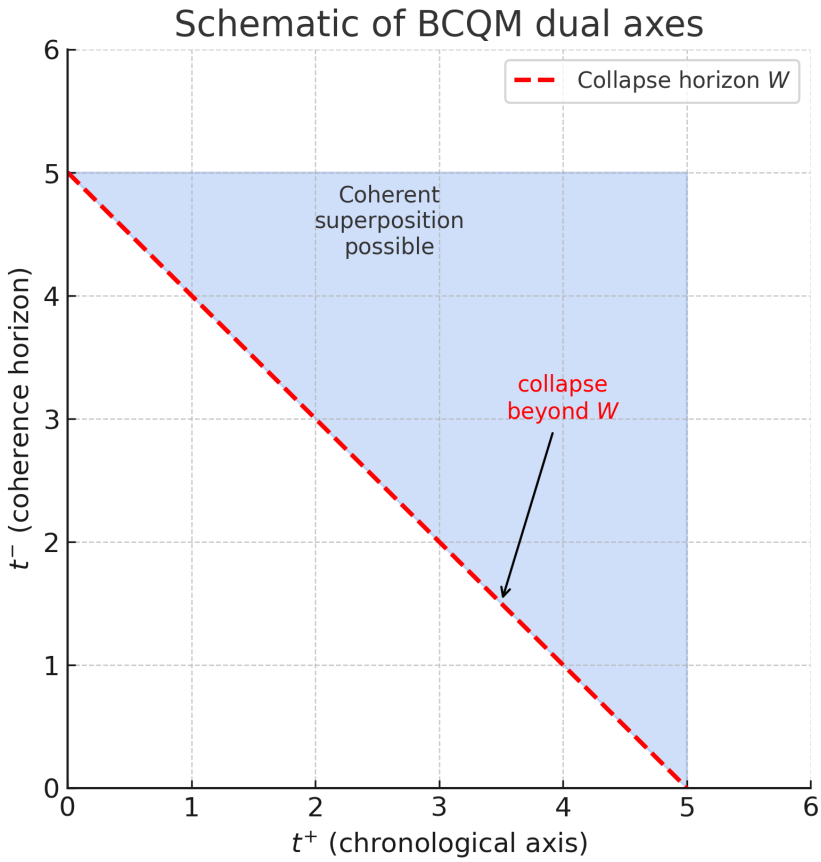

Figure 4: Schematic representation of the dual time axes (t^+ and t^-) and the horizons W (collapse).

7.4 Role of the model

The program model is illustrative only. It demonstrates how collapse can be visualised as stochastic landings in a lattice of p-wave paths, bounded by W and optionally by V. The underlying definitions and postulates remain primary; the program provides intuition and a concrete link between equations and figures.

As in standard quantum mechanics, reproducibility is an ensemble property. Repeated runs of the program model yield different stochastic landings, but the aggregate distribution of outcomes converges, over repeated runs, to the quadratic $|a_j|^2$ distribution associated with the q-wave amplitudes. This is formally identical to the Born rule; here the match is to the *applied* modulus-square weighting (motivated by pairing), whose derivation is provided in *Analytical Proofs for BCQM* [1].

8 Collapse Dynamics

8.1 Purpose and scope

This section sets out the operational rules of collapse in BCQM, showing how the q-wave probability structure leads to realised outcomes for the particle.

8.2 Triggering collapse

The q-wave itself does not collapse; it continues to encode all p-wave paths as a probability structure. What changes at the horizon W is that rollback of coherence becomes impossible. Before W, interference can be restored if records are erased. After W, this is no longer possible and collapse into a single outcome is inevitable. While each collapse realises a single outcome, only through repeated runs does the ensemble of outcomes reproduce the quadratic $|a_j|^2$ distribution. Thus repeated runs *recover the applied* modulus-square (Born) weighting.

8.3 Process of collapse

Collapse is reached when rollback is no longer possible, and one p-wave path will stochastically be realised, weighted by the q-wave amplitudes. The particle is always in spacetime as the carrier of conserved quantities such as energy, momentum, and charge. Between emission and collapse its travel is treated as a discontinuous jump, guided only by the q-wave probabilities. The q-wave itself does not undergo change in this process.

8.4 Entangled collapse as joint channel realisation

For entangled systems, crossing W realises a *joint* p-wave channel (α, β), sampled according to the joint q-wave weights $|c_{\alpha\beta}|^2$. This realisation is always constrained by conserved totals (for example $q_\alpha + q_\beta = Q_{\text{tot}}$). Operationally, a local boundary on A steers B's conditional outcomes only through post-selection, not through signalling: the marginal state of B remains unchanged without classical communication. In this way, BCQM reproduces Bell/Tsirelson correlations while keeping conservation explicit at the joint level. This formulation emphasises that conservation applies strictly to the joint system: totals are fixed by symmetry, while the distribution across subsystems remains indeterminate until collapse at W realises a specific joint outcome.

8.5 Conservation and locality

Collapse respects all conservation laws and remains consistent with no-signalling. The framework does not conflict with Bell inequalities [7] or Tsirelson limits [6], and irreversibility is imposed only by the one-way boundary condition at W, not by altering the underlying symmetric dynamics.

8.6 Empirical alignment

The timescale W is measurable and provides the empirical anchor of BCQM. In circuit-QED experiments such as [8], mid-flight durations of order microseconds align with this horizon. By symmetry a re-coherence horizon V is proposed, supported by recovery dynamics but speculative in status. Program-model simulations illustrate collapse as stochastic landings on a lattice of p-wave paths.

8.7 Summary rule

Collapse in BCQM occurs once the coherence horizon W has been exceeded. At this point rollback of interference is no longer possible, and one p-wave path will stochastically be realised. The q-wave continues to encode all channels until the outcome is recorded.

Note on V. A possible re-coherence horizon V can be introduced by symmetry as a partner to W. The idea is that just as W marks the point beyond which rollback is impossible, V would represent the point beyond which re-coherence is inevitable. In practice, V would correspond to the re-emergence of interference under controlled conditions. It is important to emphasise, however, that V is speculative. BCQM remains fully consistent without it; its role here is as a natural extension for discussion and as a pointer to possible future experiments. None of the core results of the framework depend on its inclusion.

9 Conclusion

Boundary-Condition Quantum Mechanics (BCQM) has been presented as a minimal extension of standard quantum mechanics that retains both retarded (t^+) and advanced (t^-) solutions of the Schrödinger equation. This step respects the underlying time symmetry of the dynamics and is sufficient to generate novel and testable structure.

The coexistence of t^+ and t^- solutions leads naturally to the definition of the collapse horizon W, which marks the boundary beyond which collapse is irreversible. Collapse is described not as an ad hoc rule but as the stochastic realisation of a single p-wave path at W. Repeated runs then yield ensemble statistics that recover the applied quadratic $|a_j|^2$ law—formally the Born rule. The derivation is external (Analytical Proofs), and the $t^+ - t^-$ pairing serves as the physical motivation for the modulus-square form.

BCQM is conservative in its predictions: it reproduces interference, entanglement correlations, and Bell/CHSH bounds, while remaining consistent with Tsirelson's limit and Sorkin's hierarchy. Its novelty lies in providing an explicit collapse horizon while applying the quadratic $|\psi|^2$ weighting (motivated by $t^+ - t^-$ pairing); the derivation is provided externally in Analytical Proofs. These features yield a framework that is both falsifiable and aligned with classical irreversibility.

A central outcome is that we apply the $|\psi|^2$ weighting (motivated by $t^+ - t^-$ pairing), with the derivation provided in Analytical Proofs; ensemble behaviour matches this applied law. This demonstrates that ensemble frequencies match the *applied* modulus–square (Born) law; the *derivation* is provided externally in *Analytical Proofs for BCQM* [1].

A defining falsifier of BCQM is that interference visibility drops sharply once path separation exceeds the collapse horizon W, even at zero temperature, and this loss cannot be undone. Standard quantum mechanics predicts no such intrinsic floor: visibility is limited only by environmental noise. This distinction makes W a direct experimental handle for testing BCQM against conventional theory (see Appendix D).

Experimental proposals. The framework yields clear experimental proposals that distinguish BCQM from standard quantum mechanics:

- **Rollback window:** collapse can be reversed if interrupted before the horizon W, but not once W is exceeded.
- **Visibility floor:** interference visibility drops sharply once path separation exceeds W, even at zero temperature, and this loss cannot be recovered by environmental isolation.
- **Re-coherence test:** a speculative horizon V would allow re-emergence of superposition after a void interval; no such effect is predicted by standard theory.

These proposals provide concrete handles for falsifying BCQM in controlled settings, directly extending the empirical anchor provided by circuit-QED experiments such as Minev *et al.*

In summary, BCQM makes three positions clear. First, the stochastic growth uses the quadratic law as *implemented* here; its *derivation* is external [1]. The t^+–t^- pairing provides the physical motivation for the modulus-square form. Second, including both roots underpins the time-symmetric construction of the q-wave while preserving operational consistency with standard quantum mechanics. Third, the proposed re-coherence horizon V is speculative and not required for the main results.

Future directions include exploring experimental tests of collapse timescales, rollback probabilities near W, and the possible signatures of re-coherence processes.

9.1 Code availability

The simulation programs used to generate figures and results in this work are openly available at [9]

https://github.com/PMF57/BCQM-Programs.

Appendix

A Consistency Checks

BCQM extends standard quantum mechanics by introducing the q-wave and finite collapse horizons. It is essential that the framework remains consistent with established no-go theorems and operational bounds. This appendix summarises the checks performed.

A.1 No-signalling

BCQM respects the no-signalling principle. The q-wave encodes probabilities but carries no conserved quantities and transmits no information. Collapse occurs only locally, with the particle carrying conserved quantities at emission and collapse. As a result, marginal statistics at one site are independent of measurement settings at another, consistent with relativistic causality. In particular, retaining the advanced TDSE branch does not enable retrocausal influences: reduced states and marginal statistics remain independent of future settings without classical communication.

A.2 Bell and Tsirelson bounds

BCQM reproduces the correlations of standard quantum mechanics. The stochastic realisation of p-wave channels is governed by the q-wave amplitudes, which follow the same Born-rule weighting as in orthodox QM [10]. Consequently, Bell inequalities can be violated up to the Tsirelson bound $2\sqrt{2}$ [6] but never exceeded. No hidden-variable mechanism is introduced.

A.3 Entanglement correlations

For a pair of spin-$\frac{1}{2}$ particles prepared in the singlet state, the BCQM treatment of measurements yields the same correlation function as standard quantum mechanics:

$$E(\mathbf{a}, \mathbf{b}) = -\cos\theta_{\mathbf{ab}}.$$

In BCQM the singlet enforces conservation of the total spin, while the allocation to individual subsystems (up or down for each particle) remains indeterminate until collapse. The joint q-wave ensures that whichever outcome is realised, the total constraint is always satisfied. Here $\theta_{\mathbf{ab}}$ is the angle between detector settings $\mathbf{a}$ and $\mathbf{b}$. The corresponding CHSH parameter then satisfies

$$|S| \leq 2\sqrt{2},$$

preserving the Tsirelson bound. The horizon W affects only the persistence of visibility over time, not the maximum attainable correlations.

A.4 Sorkin hierarchy

The Sorkin parameter I_3 [11] tests for higher-order interference beyond quantum mechanics. In BCQM the q-wave remains a linear superposition of TDSE solutions, and Fourier decomposition preserves linear additivity of channel weights. Thus $|I_3| = 0$ as in standard QM, and no third-order interference is predicted.

A.5 Loschmidt reversibility

The underlying TDSE dynamics are time-symmetric. Irreversibility in BCQM arises only from the one-way boundary at the coherence horizon W: once W is passed, rollback of interference is impossible. This preserves consistency with Loschmidt's paradox [12]: microscopic dynamics remain reversible, while macroscopic irreversibility emerges from boundary conditions.

A.6 Poincaré recurrence

In a finite Hilbert space, unitary evolution implies recurrences. BCQM modifies this by imposing collapse once W is exceeded, preventing indefinite evolution of superpositions. Recurrence of macroscopic records is therefore operationally suppressed, consistent with observation. At the mathematical level, the q-wave remains intact, but realised outcomes are fixed by collapse and cannot be undone.

A.7 Summary

Across all tests, BCQM reproduces the operational limits of quantum mechanics: no-signalling is preserved, Bell violations [7] are bounded by Tsirelson [6], Sorkin's I_3 [11] vanishes, and irreversibility is consistent with Loschmidt [12] and Poincaré recurrence [13]. The framework therefore aligns with known consistency requirements while extending collapse dynamics via the finite horizon W.

B GKLS generators and the coherence horizon W

Consider the GKLS master equation

$$\dot{\rho} = \mathcal{L}[\rho] = -\,i[H,\rho] + \sum_j \Big(L_j \rho L_j^\dagger - \tfrac{1}{2}\{L_j^\dagger L_j, \rho\} \Big)\,, \tag{10}$$

with eigenvalues $\{\lambda_k\}$. The real parts of nonzero λ_k give dissipative rates. The coherence horizon W is defined as

$$W = \frac{1}{\Delta}, \qquad \Delta = \min\{-\Re\lambda_k : \lambda_k \neq 0\}. \tag{11}$$

B.1 Toy models

B.1.1 Amplitude damping

With $L = \sqrt{\gamma}\ |g\rangle\langle e|$, the off–diagonal decays as

$$\rho_{eg}(t) = \rho_{eg}(0)\, e^{-(\gamma/2+\Gamma_\phi)t}. \tag{12}$$

Applying the threshold rule $C(W) = \varepsilon$ gives

$$W = \frac{1}{\gamma/2 + \Gamma_\phi}\ \ln\!\left(\frac{C(0)}{\varepsilon}\right). \tag{13}$$

B.1.2 Pure dephasing

With $L = \sqrt{\gamma_\phi}\,\sigma_z$, populations are conserved and

$$\rho_{eg}(t) = \rho_{eg}(0)\, e^{-2\gamma_\phi t}. \tag{14}$$

Applying the threshold rule gives

$$W = \frac{1}{2\gamma_\phi}\ \ln\!\left(\frac{C(0)}{\varepsilon}\right). \tag{15}$$

Note that here $2\gamma_\phi$ plays the role of Γ_2.

B.2 Spectral-gap rule

More generally,

$$W = \frac{1}{\Delta} \ln\left(\frac{C(0)}{\varepsilon}\right), \qquad \Delta = \min\{-\Re\lambda_k : \lambda_k \neq 0\}. \tag{16}$$

This expression shows that W is set by the smallest non-zero real part of the GKLS spectrum.

B.3 Ramsey illustration

In a Ramsey experiment, coherence visibility decays as $V(t) = \exp(-t/T_2)$, where T_2 is the transverse relaxation time. Identifying W with T_2 through the threshold rule yields results consistent with circuit-QED measurements [8]

B.4 Arrow of time connection

The GKLS generator $\mathcal{L}_H$ induces a contractive semigroup on states and, in the Heisenberg picture, on effects. For any two states ρ_1 and ρ_2, the relative entropy is monotone under the map:

$$D(\rho_1 \| \rho_2) \geq D(\Phi_t[\rho_1] \| \Phi_t[\rho_2]).$$

This monotonicity implies that distinguishability, and therefore interference contrast, cannot increase with elapsed time. The behaviour mirrors Boltzmann's H-theorem, where entropy H increases irreversibly. In BCQM, the finite horizon W gives this contractivity a direct operational meaning: once superposition visibility is lost, it cannot be recovered by local operations.

C Expanded Ramsey Example

C.1 Setup

Consider a two-level system with basis $\{|0\rangle, |1\rangle\}$, energy splitting $\hbar\omega_0$, and Hamiltonian

$$H = \frac{\hbar\omega_0}{2}\sigma_z. \tag{17}$$

The system is initialised in a superposition by a $\pi/2$ pulse:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \tag{18}$$

C.2 Open-system dynamics

The qubit undergoes amplitude damping at rate Γ and pure dephasing at rate γ_ϕ. The GKLS equation is

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \tfrac{\Gamma}{2}\mathcal{D}[\sigma_-]\rho + \gamma_\phi \mathcal{D}[\sigma_z]\rho, \tag{19}$$

where $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$. The off-diagonal element evolves as

$$\rho_{01}(t) = \rho_{01}(0)\, e^{-(\Gamma/2+\gamma_\phi)t}\, e^{-i\omega_0 t}. \tag{20}$$

C.3 Ramsey signal

After free evolution for time t, a second $\pi/2$ pulse maps coherence into population. The measured Ramsey signal is

$$P(t) = \tfrac{1}{2}\left[1 + e^{-(\Gamma/2+\gamma_\phi)t}\cos(\omega_0 t)\right]. \tag{21}$$

The visibility of the fringes decays with time constant

$$T_2^* = \frac{1}{\Gamma/2 + \gamma_\phi}. \tag{22}$$

C.4 Collapse horizon

In BCQM the coherence horizon W is identified with this characteristic decay time, up to a threshold convention. For visibility $V(t) = e^{-(\Gamma/2+\gamma_\phi)t}$, the condition $V(W) = e^{-1}$ gives

$$W = \frac{1}{\Gamma/2 + \gamma_\phi}. \tag{23}$$

This matches the standard T_2^* of Ramsey interferometry, providing a direct experimental anchor for W.

C.5 Discussion

The Ramsey experiment therefore illustrates concretely how W arises from irreversible decoherence dynamics. The q-wave encodes both retarded and advanced solutions, but once $t > W$ rollback is no longer possible and collapse becomes inevitable. Experimental data from superconducting qubits confirm this timescale at the level of a few microseconds.

D Empirical Anchors for W and V

D.1 Minev et al. (2019)

In superconducting circuit-QED systems, Minev et al. (2019) [8] monitored quantum jumps in real time. Their experiment tracked a three-level artificial atom with states $\{|g\rangle, |e\rangle, |f\rangle\}$, using continuous weak measurement to resolve the onset and reversal of jumps. A key finding was that mid-flight coherence can be monitored and, if interrupted early, reversed.

The mean jump duration was observed to be $\sim 4\,\mu$s. This provides an empirical estimate for the coherence horizon W: rollback of interference was possible only for durations shorter than this. Once the mid-flight duration exceeded this timescale, collapse was irreversible.

Although the Minev *et al.* circuit-QED results provide the clearest indication of a finite collapse horizon, BCQM does not rely on this platform alone. Comparable coherence–decay signatures and rollback phenomena are accessible in other architectures, including trapped ions and nitrogen-vacancy centres in diamond. These systems offer long coherence times, high-fidelity control, and single-shot detection, making them suitable candidates for independent estimation of W and for testing the universality of the horizon. In this way, BCQM links naturally to multiple experimental domains rather than a single technological implementation.

D.2 Recovery dynamics

The same study showed that recovery of coherence was possible after partial collapse. Recovery times were significantly shorter, typically 0.3–0.5 μs. By symmetry, BCQM interprets these as evidence of a re-coherence horizon V, mirroring the role of W on the recovery side. While W has direct theoretical grounding in GKLS eigenvalues, V is introduced as a phenomenological horizon supported by these observations.

D.3 Other platforms

Comparable coherence times are reported in other qubit platforms, such as trapped ions and NV centres in diamond, though with parameter-dependent scaling. In each case, a finite coherence time can be identified that plays the role of W. Systematic investigation of recovery dynamics across these platforms could test the universality of V.

D.4 Summary

Empirical data support the existence of a finite coherence horizon W of order microseconds in circuit-QED systems. Evidence from recovery dynamics further suggests a re-coherence horizon V at shorter timescales. Together these provide experimental anchors for the temporal boundaries postulated in BCQM.

E Program Model Methods and Reproducibility

E.1 Simulation design

The program model is implemented as a Galton-style lattice of branches. Each branch corresponds to a p-wave channel derived from Fourier decomposition of the q-wave. The particle's collapse is simulated as a stochastic landing at one of the lattice leaves. Landings are sampled according to the channel weights, which evolve under TDSE and GKLS dynamics.

E.2 Algorithm

At each timestep:

1. Initialise channel weights from the q-wave amplitudes.
2. Evolve weights according to the unitary Hamiltonian and GKLS dissipators.
3. Apply stochastic sampling to realise a channel when the horizon W is exceeded.
4. Record the landing outcome.

Rollback of interference is allowed for $t < W$, suppressed for $t > W$, consistent with the definition of the coherence horizon.

E.3 Figures

The main text presents example outputs:

- Channel lattice schematic (Fig. 1).
- Distribution of channel weights (Fig. 2).
- Collapse enforced beyond W (Fig. 3).
- Dual-axis schematic with W and V (Fig. 4).

E.4 Ensemble behaviour

Individual runs yield different stochastic landings. Repeated runs build up an ensemble histogram of outcomes. As in standard quantum mechanics, reproducibility is achieved only at the ensemble level: the aggregate distribution matches the modulus-square (Born) probabilities set by the q-wave amplitudes [10]. Figure 5 shows an example histogram compared with incoherent expectation.

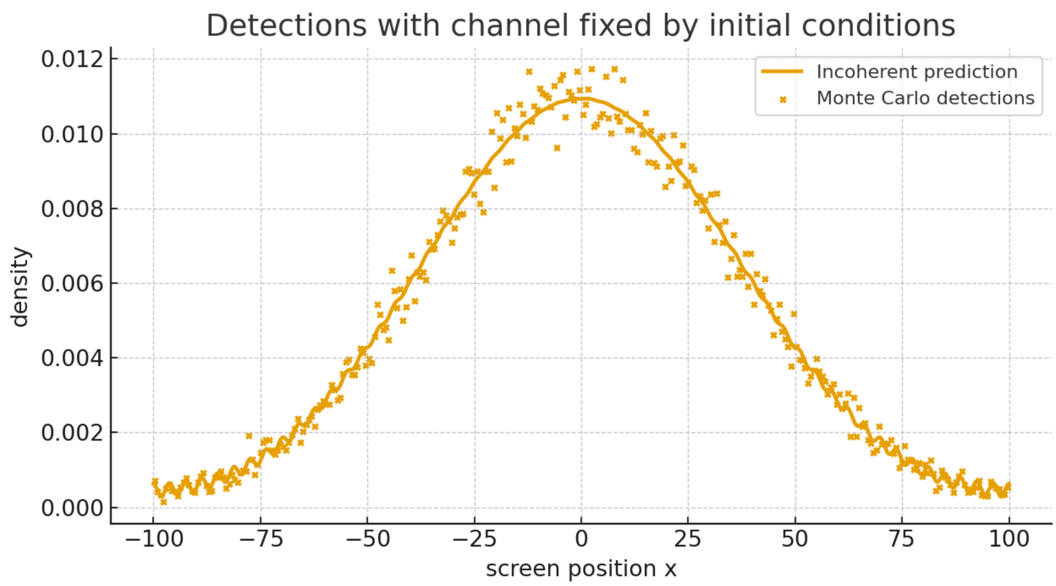

Figure 5: Histogram of outcomes from repeated program model runs compared with incoherent baseline. Ensemble reproducibility matches the modulus-square (Born) weights [10].

E.5 Summary

The program model demonstrates how BCQM collapse can be represented as stochastic landings in a lattice of p-wave channels. While individual outcomes vary, ensemble statistics reproduce the modulus-square (Born) weighting [10], ensuring reproducibility.

F Born Weighting Revealed by Ensemble Statistics

Scope. This appendix visualizes how ensemble frequencies recover the modulus–square weighting used in the main text.
Provenance. We *apply* the $|\psi|^2$ weighting, *motivated* by pairing the retarded amplitude with its conjugate; a derivation under explicit axioms is provided in *Analytical Proofs for BCQM* [1].
Ensembles. The figures below show that repeated realisations reproduce this applied modulus–square law; no additional rule is introduced beyond this.

F.1 Interference versus incoherent sum

Figure 6 shows the baseline contrast between a coherent q-wave interference pattern and the incoherent sum of channels under full damping. Only the coherent case displays interference fringes, while the incoherent sum yields a smooth distribution. This establishes the reference against which collapse is tested.

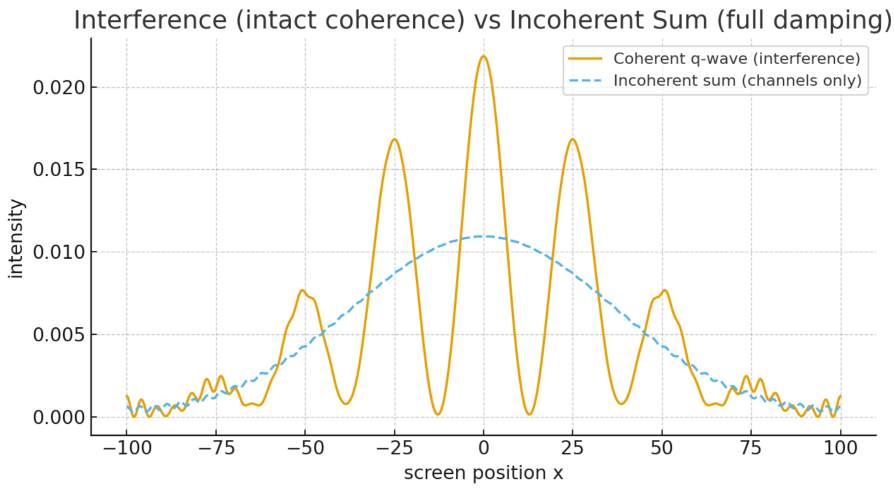

Figure 6: Coherent q-wave interference compared with the incoherent sum (full damping).

F.2 Finite W as collapse damping

Figure 7 demonstrates that a finite horizon W progressively suppresses interference, interpolating between full coherence and incoherent channel addition. This matches the role of collapse as implemented in the model.

Figure 7: Finite horizon W suppresses interference, producing intermediate visibility.

F.3 Probability channels

Figure 8 shows the decomposition into probability channels. The incoherent sum of channel contributions matches the envelope of the damped pattern, providing the link between q-wave amplitudes and outcome probabilities.

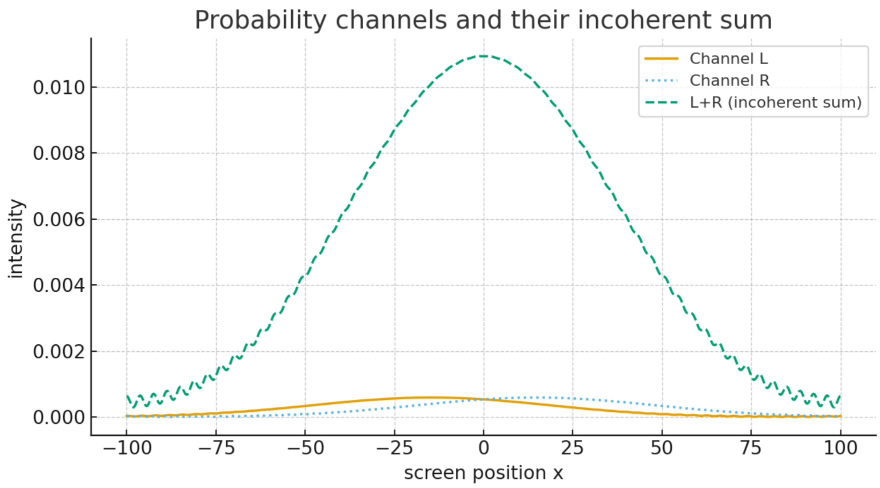

Figure 8: Probability channels (L,R) and their incoherent sum. This is the same schematic as Fig. 2, reproduced here to emphasise the role of channels in ensemble behaviour.

F.4 Ensemble frequencies

Monte Carlo runs of collapse ticks select channels stochastically, weighted by q-wave amplitudes. Figure 9 compares the resulting histogram of detections with the incoherent prediction, showing close alignment. No additional rule beyond the applied modulus–square weighting is used; the match arises from repeated runs (law of large numbers).

Figure 9: Monte Carlo detections compared with incoherent prediction. Ensemble behaviour realises the modulus–square (Born) distribution.

F.5 Discrete outcome lattice

Finally, Figure 10 shows the lattice of discrete landing sites, revealing the underlying outcome structure. Frequencies across many runs converge to the modulus–square (Born) distribution, completing the link from q-wave to particle detection.

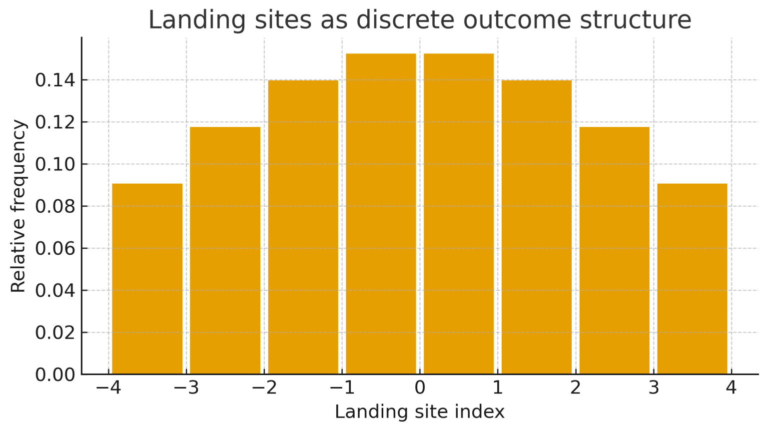

Figure 10: Discrete landing site structure. Frequencies converge to modulus–square (Born) distribution.

F.6 Summary

Together, these outputs demonstrate that in BCQM we *apply* the modulus–square weighting (motivated by pairing), and ensembles *recover* this applied law. The figures illustrate the transition from coherent interference to the incoherent envelope under finite W, and the consistency of repeated realisations with the $|\psi|^2$ distribution justified in *Analytical Proofs for BCQM* [1].

G Algorithmic Addendum: Stochastic Event-Realisation Engine

Purpose. This addendum fixes the exact sampling law used by the simulator. At step n, given candidates $x \in \mathcal{X}_n$ and the retarded kernel $K_r^{(+)}(x \mid E_n)$ defined in §"Local kernels", we form the nonnegative weights $w(x) = |K_r^{(+)}(x \mid E_n)|^2$ and *apply* the normalized quadratic law

$$\mathbb{P}(E_{n+1} = x \mid E_n, \mathcal{X}_n) \;=\; \frac{w(x)}{\sum_{y \in \mathcal{X}_n} w(y)}.$$

This modulus–square weighting is *motivated* by pairing the retarded amplitude with its conjugate; a derivation under explicit axioms is provided in *Analytical Proofs for BCQM* [1]. Device asymmetries and small drifts belong in the real functional $F[\gamma; W, w_{\text{env}}]$ that builds $K_r^{(+)}$ (gain/attenuation), not in hand-written channel probabilities.

Foundational wording (BCQM). The q-wave is a probability/propensity structure only; conserved quantities belong to the particle. *Collapse = absorption of the q-wave.* Exactly one outcome is *realised* (not "selected") at each step, with realisation probabilities induced by the q-wave amplitudes.

State, inputs, and parameters

1. **Realised state.** A finite sequence of realised events $\{E_0, E_1, \ldots, E_n\}$; E_n is the current tip.
2. **Candidate set.** A finite set $\mathcal{X}_n$ of kinematically allowed candidate events x in the next step. This is generated by a hop-bound (radius r in the underlying graph/geometry) and a local window in proper time controlled by W (see below).
3. **Edge amplitudes.** Complex edge-amplitudes $a(e)$ attached to directed edges e; path amplitudes multiply along a path.

4. **Windows and scales.** Coherence horizon $W > 0$ (proper-time scale); hop radius r (a positive integer); optional environment window w_{env} for modelled dephasing/absorption by apparatus or bath.

5. **Measurement channels (optional).** A partition $\{C_j\}$ of $\mathcal{X}_n$ induced by a device; each C_j corresponds to a macroscopic record channel (pixel, port, pointer sector, etc.).

6. **RNG seed.** A fixed seed s for reproducibility.

Local kernels, t^+–t^- pairing, and weights

Given E_n and candidates $x \in \mathcal{X}_n$, define the *retarded (forward, t^+) kernel*

$$K_r^{(+)}(x \mid E_n) \;=\; \sum_{\gamma: E_n \to x,\; \ell(\gamma) \le r} \left[\prod_{e \in \gamma} a(e)\right] F[\gamma; W, w_{\text{env}}], \tag{24}$$

where the sum runs over paths γ of at most r hops from E_n to x, and $F[\gamma; W, w_{\text{env}}] \in \mathbb{C}$ is a window factor that enforces the finite coherence horizon and any local dephasing/absorption.

The *advanced (coherence, t^-) co-kernel* is taken as the complex-conjugate dual at the candidate event,

$$K_r^{(-)}(x \mid E_n) \;:=\; \left(K_r^{(+)}(x \mid E_n)\right)^*, \tag{25}$$

which encodes the BCQM time-symmetric pairing at the meeting point x without implying backwards-in-time dynamics.[2]

The *propensity weight* at x is the t^+–t^- pairing:

$$w(x) \;=\; K_r^{(+)}(x \mid E_n)\, K_r^{(-)}(x \mid E_n) \;=\; \left|K_r^{(+)}(x \mid E_n)\right|^2. \tag{26}$$

Realisation probability (applied law).

$$\mathbb{P}(E_{n+1} = x \mid E_n, \mathcal{X}_n) \;=\; \frac{w(x)}{\sum_{y \in \mathcal{X}_n} w(y)} \;=\; \frac{\left|K_r^{(+)}(x \mid E_n)\right|^2}{\sum_{y \in \mathcal{X}_n} \left|K_r^{(+)}(y \mid E_n)\right|^2}. \tag{27}$$

Provenance. We *apply* the modulus–square law in this paper, motivated by t^+–t^- pairing; the *derivation* under explicit axioms is provided in *Analytical Proofs for BCQM* [1]. No softmax or additional reweighting is introduced beyond the normalization in (27). Apparatus asymmetries enter only through $F[\gamma; W, w_{\text{env}}]$ when constructing $K_r^{(+)}$.

Categorical draw and realisation

Form the normalised distribution over candidates

$$p(x) \;=\; \frac{w(x)}{\sum_{y \in \mathcal{X}_n} w(y)}. \tag{28}$$

Draw one outcome $X \sim \text{Categorical}(p)$ using the RNG seeded by s. *Exactly one* outcome is realised: set $E_{n+1} := X$ and append to the realised history.

[2] In BCQM, t^- parameterises the coherence axis and is *not* literal backward time. The window W bounds how far the advanced contribution can coherently pair with the retarded one.

Absorption update (collapse) and record formation

1. **Absorption.** Treat the realisation as absorption of the q-wave into E_{n+1}: amplitudes incompatible with E_{n+1} are removed from further propagation (their contribution to future kernels is zero).

2. **Record logic (apparatus).** If $E_{n+1} \in C_j$ and the apparatus is modelled with gain g_j, then when the record variable crosses its amplification threshold the process becomes effectively irreversible (subsequent weights outside C_j are suppressed to numerical zero by F). This models detection without inserting any post hoc reweighting.

Candidate generation and windows

1. **Hop-bounded kinematics.** Construct $\mathcal{X}_n$ as the set of kinematically allowed nodes reachable in at most r hops from E_n (respecting any light-cone support in the amplitude assignment).

2. **Horizon filter.** Discard candidates for which all contributing paths have $\Delta\tau(\gamma) \gg W$; practically, this occurs automatically via the factor F.

3. **Environment filter (optional).** Apply device/bath-specific attenuation inside F through $\eta(\gamma; w_{\mathrm{env}})$.

Minimal pseudocode (package-free)

```
# Inputs: E = [E0,...,En], graph G with edge amplitudes a(e), hop radius r,
#         horizon W, env window w_env (optional), RNG seed s
# Output: Extended realised history E' = [E0,...,En, En+1]

set_seed(s)
X_candidates = gen_candidates(E_n, G, r)  # hop-bounded, kinematically allowed
weights = []
for x in X_candidates:
    K = 0
    for path in paths(E_n -> x, length<=r, in G):
        amp = 1
        for e in path:
            amp *= a(e)
        F = horizon_window(path, W) * env_window(path, w_env)
        K += amp * F
    weights.append( abs(K)**2 )  # arises from t+ t- pairing at x
probs = normalize(weights)
En_plus_1 = categorical_draw(X_candidates, probs)  # one outcome realised
E.append(En_plus_1)
apply_absorption_update(E, En_plus_1)  # zero incompatible branches / records
return E
```

Reproducibility and audit checklist

1. **Seeding.** Expose the RNG seed and record it with every run.

2. **Weights log.** Log $K_r^{(+)}(x \mid E_n)$ and $w(x)$ for all $x \in \mathcal{X}_n$ at each step before the draw.

3. **No covert Born step.** Search the codebase to ensure no additional squaring or manual probability injection occurs beyond the $|\cdot|^2$ in Eq. (26). Typical audit: grep for "born", "prob_inject", and extra "**2" on amplitude-like variables.
4. **Device modelling.** Any detection asymmetry must enter only via $a(e)$ or the window factor F (gain, attenuation), not via hand-written channel probabilities.
5. **Limits.** Check $W \to \infty$ (full-coherence) and strong-apparatus (fast absorption) limits to verify expected behaviours.

Notes on the double-slit (sanity check)

In a two-slit set-up, $K_r^{(+)}$ is the sum of two path families meeting at the screen point x. The weight $w(x) = |K_r^{(+)}(x)|^2$ contains the interference term automatically from the t^+–t^- pairing; no Born rule is applied anywhere else.

Scope (other programs). The same weighting rule—pairing the forward t^+ and coherence/advanced t^- contributions to yield $w = |K|^2$—is what is implemented in `bcqm_double_slit*.py` and `bcqm_toy_experiment.py`. For `ramsey.py`, outcome probabilities are $p_j = |\langle j|\psi_{\rm out}\rangle|^2$ (the same pairing at the final projection), and for `gkls.py`, when a measurement is sampled it is $p_j = \mathrm{Tr}(P_j\rho)$ (pairing encoded in ρ). The provided `galton.py` is a classical demo (no complex amplitudes), so this addendum does not apply to it.

Takeaway. The algorithm applies the modulus-square law by sampling from weights w(x) = $|K^{(+)}(x)|^2$. This rule is motivated by the $t^+ - t^-$ pairing, and its formal derivation is provided in Analytical Proofs for BCQM [1]. The algorithm's implementation is transparent and auditable, with no covert application of other probability rules.

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