Boundary-Condition Quantum Mechanics IV: Inertial Noise, Universality, and Entangled Clusters

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Abstract

BCQM IV builds on the Boundary-Condition Quantum Mechanics programme developed in BCQM I–III [1, 2, 3] and the Primitives [4] note by working entirely in the emergent spacetime regime. Starting from the hop-bounded stochastic growth model for Events introduced in BCQM III [3], we ask what *irreducible fluctuations* remain around the emergent inertial motion of a realised thread once ordinary environmental noise is idealised away. We define *inertial noise* as the residual stochastic departure of realised trajectories from the BCQM inertial drift, and characterise it via the power spectral density of velocity or acceleration.

We then show, for a broad class of local graph-phase rules with a finite coherence horizon W_{coh}, that the inertial noise spectrum has a universal structure: its shape and scaling with W_{coh} are fixed by the same effective relativistic action that governs the deterministic drift, while only a small number of dimensionless parameters remain model-dependent. These parameters capture, for example, details of the local graph connectivity (such as coordination number and mild anisotropy) and the short-range temporal correlations of the graph-phase noise, and are expected to take values of order unity. This universality is justified by the "graphs to invariant action" continuum-limit programme outlined in the companion BCQM technical note on emergent inertia and invariant action [5].

Finally, we extend the BCQM primitives to *entangled clusters*. We define a configuration-space Event graph for clusters that reproduces the BCQM II [2] joint q-wave law on $\mathcal{H}_A \otimes \mathcal{H}_B$ without enabling signalling, and we formulate a locality notion based on hop-bounded moves in configuration space. Within this primitive we track how the inertial noise of a cluster's centre-of-mass arises from the same graph-based mechanism as for single threads. The resulting inertial-noise floor is proposed as a falsifiable consequence of BCQM, with numerical experiments and possible observational signatures outlined in the final sections.

1 Introduction and roadmap

The Boundary-Condition Quantum Mechanics (BCQM) framework treats the q-wave as an informational field that constrains a single realised particle thread along the ordinary chronological axis t^+, while a finite coherence horizon W_{coh} encodes the recoverability of quantum interference along the coherence axis t^-. BCQM I [1] introduced this two-axis picture and the collapse horizon as a boundary-condition modification of standard quantum mechanics. BCQM II [2] and the Primitives note [4] then recast spacetime itself as an emergent causal graph of irreversible Events generated by a local, hop-bounded selection rule on an underlying amplitude-weighted graph, with smooth relativistic dynamics appearing as a coarse-grained description in suitable regimes.

BCQM III [3] supplied the concrete stochastic growth engine for this picture by specifying a hop-bounded rule for adding Events to the realised graph. In the appropriate emergent regime, the average motion of a realised thread obeys an effective inertial law with a mass parameter $m_{\text{eff}}(W_{\text{coh}})$ determined by the same structure that defines the collapse horizon. In other words, the existence of a finite recoverability horizon not only gives an arrow of time but also fixes the inertial response of realised trajectories.

The present paper asks what *stochastic residue* is left once this deterministic inertial drift has been accounted for. Idealising away ordinary environmental noise, we define *inertial noise* as the irreducible fluctuations of realised worldlines around the BCQM inertial trajectory generated by the BCQM III [3] engine. Our first goal is to characterise this inertial noise in terms of a power spectral density for velocity or acceleration, and to express its scaling behaviour in terms of the coherence horizon W_{coh}.

Our second goal is to show that this inertial noise spectrum is *universal* across a broad class of microscopic graph-phase rules. Here we rely on the "graphs to invariant action" continuum-limit programme outlined in the BCQM technical note on emergent inertia and invariant action [5], which argues that local, phase-covariant, hop-bounded rules with modest homogeneity assumptions lead in the emergent spacetime regime to trajectories governed by an effective relativistic action. We take that result as a structural input and demonstrate that, under the same conditions, the form of the inertial noise spectrum is fixed up to a small set of dimensionless parameters.

The third goal is to extend the BCQM primitives from single threads to *entangled clusters*. We introduce a configuration-space Event graph for clusters (built either as a product graph or a restricted subgraph) that reproduces the BCQM II [2] joint q-wave law on $\mathcal{H}_A \otimes \mathcal{H}_B$ while preserving no-signalling and conservation laws. Within this framework we analyse how the centre-of-mass motion of a cluster inherits inertial noise from the underlying event process.

Interpretation note (no retrocausality). Throughout this paper we work entirely in the emergent spacetime regime, where the effective relativistic action already summarises the underlying two-axis amplitude bookkeeping. Any reference to an "advanced" branch remains purely mathematical—the advanced Green's-function contribution used to keep the amplitude description time-symmetric—and is not interpreted as physical evolution backwards in time. Causes, records, and interventions remain ordered along the ordinary laboratory time axis t^+.

Technical note. The detailed derivations of the proper-time inertia model and the graph-to-action continuum-limit programme are collected in the BCQM technical note on emergent inertia and invariant action [5]; here we summarise only the structural consequences needed for BCQM IV.

The structure of the paper is as follows. In section 2 we summarise the BCQM III growth engine and define the inertial-noise observable, expressed as a power spectral density. Section 3 develops the universality argument using the graph-phase model class and its continuum limit to an effective action. Section 4 introduces the entangled-cluster primitive in configuration space and analyses inertial noise for cluster centre-of-mass motion. In section 5 we outline numerical experiments and sketch possible observational signatures. We conclude in section 6 with a summary and a brief discussion of open questions, including dimensionality selection, fully rigorous universality proofs, and the role of gravity in BCQM V.

2 From BCQM III to inertial noise

2.1 Summary of the BCQM III engine

BCQM III provides the stochastic growth model that turns the abstract BCQM primitives into concrete realised trajectories in an emergent spacetime. The underlying ingredients are: a discrete set of candidate Events, a local hop radius r on the propensity-weighted graph, and a finite coherence horizon $W_{\rm coh}$ that bounds how far coherent amplitude contributions can remain relevant along the coherence axis t^-. At each growth step the engine evaluates a local kernel K_r on the neighbourhood of the current realised Event, with contributions from both the retarded and advanced amplitude branches, and selects the next realised Event with probability proportional to $|K_r|^2$. In this way the q-wave remains a purely informational object: it encodes the *propensities* for different growth options, while the realised particle thread is a single stochastic sample constrained by those propensities.

In the emergent spacetime regime, where the realised Event graph admits a smooth coarse-grained embedding into a Lorentzian manifold, the average motion of a single thread can be described by an effective relativistic action. The analysis in BCQM III (together with the companion BCQM technical note on emergent inertia and invariant action [5]) shows that the corresponding effective mass parameter scales as

$$m_{\rm eff} \;\propto\; W_{\rm coh}^{-2}, \tag{1}$$

up to model-dependent factors that capture details of the local kernel and environment. Intuitively, a larger coherence horizon allows amplitudes to correlate growth decisions over longer histories, reducing the randomness of the realised trajectory and thereby increasing the effective inertia. Conversely, a shorter horizon suppresses long-range correlations and yields a "lighter" response.

Even in the idealised limit where environmental noise is stripped away, the BCQM III engine never produces exactly straight, deterministic worldlines. The hop-bounded selection rule, combined with a finite $W_{\rm coh}$, always leaves a residual stochastic component in the step-to-step increments of the realised trajectory. This appears operationally as *jitter*: small, random deviations of the realised path around the coarse-grained inertial drift. Attempts at finite-time reversal—for example, by running the growth engine backwards from a coarse-grained final state—are necessarily imperfect once the Event record has accumulated beyond the coherence horizon. These two features, intrinsic jitter and finite-time reversal limitations, are the main precursors of the inertial noise floor studied in the present paper.

2.2 Definition of inertial noise observables

To make the notion of inertial noise precise we work with the realised trajectory of a single thread in the emergent spacetime description. Let the Events on this thread be labelled by an integer index n, and let x_n denote the coarse-grained spacetime position of the n-th Event in some inertial laboratory frame. We assume that along the realised thread there is a monotonically increasing laboratory time coordinate t_n associated with each Event, compatible with the emergent causal structure of the graph.

From these data we define discrete velocities and accelerations by

$$v_n \;=\; \frac{x_{n+1}-x_n}{t_{n+1}-t_n}, \qquad a_n \;=\; \frac{v_{n+1}-v_n}{t_{n+1}-t_n}. \tag{2}$$

In the regime where a large number of Events fall within any macroscopic time interval of interest, these discrete sequences can be interpolated to define continuous-time processes $v(t)$ and $a(t)$, with the understanding that they are only defined up to coarse-graining on timescales comparable to the typical inter-Event spacing.

The *inertial drift* is captured by the ensemble-averaged velocity $\mathbb{E}[v(t)]$ obtained by running the BCQM III engine many times with the same macroscopic initial conditions. In the emergent regime this drift obeys the effective equations of motion derived from the relativistic action and corresponds to motion with mass $m_{\rm eff}(W_{\rm coh})$. The *inertial noise* is then defined as the fluctuation of the realised velocity around this drift,

$$\delta v(t) \;=\; v(t) - \mathbb{E}[v(t)], \tag{3}$$

and likewise for the acceleration fluctuations $\delta a(t)$.

The primary observable we use to characterise inertial noise is the power spectral density (PSD) of the velocity fluctuations. Formally, for a long sample of duration T we define

$$S_v(\omega) \;=\; \lim_{T\to\infty} \frac{1}{T}\left| \int_0^T \delta v(t)\, e^{-i\omega t}\, \mathrm{d}t \right|^2, \tag{4}$$

or, in the discrete form appropriate to simulations with time step Δt,

$$S_v(\omega_k) \;=\; \frac{\Delta t}{N}\left| \sum_{n=0}^{N-1} \delta v_n\, e^{-i\omega_k n\Delta t} \right|^2, \qquad \omega_k = \frac{2\pi k}{N\Delta t}, \tag{5}$$

with N the total number of samples. An analogous definition applies to the acceleration PSD $S_a(\omega)$, which may be more natural for some experimental probes.

Several regimes of timescales are important. At very short times comparable to the typical inter-Event separation, the discreteness of the Event graph dominates and the continuum approximation implicit in the PSD definition breaks down. At very long times, ordinary environmental noise and external forces will typically dominate over the intrinsic BCQM contribution. The window of interest for BCQM IV therefore lies in an intermediate regime

$$\Delta t_{\text{Event}} \;\ll\; T \;\ll\; T_{\text{env}}, \tag{6}$$

where Δt_{Event} is the average time between successive Events on the realised thread and T_{env} characterises the timescale on which external noise sources or environment-induced drift become significant.

Within this window the inertial noise extracted from $S_v(\omega)$ or $S_a(\omega)$ can be attributed to the intrinsic stochasticity of the BCQM growth engine itself. The central claim of this paper is that, once expressed in terms of the coherence horizon W_{coh} and the effective mass $m_{\text{eff}}(W_{\text{coh}})$, the shape and scaling of this intrinsic spectrum are universal across a broad class of underlying graph-phase rules. The supporting arguments and explicit scaling laws are developed in the next section, where we connect the BCQM III engine to the continuum "graphs to invariant action" programme.

3 Universality and continuum limits

3.1 Graph-phase model class

The BCQM III growth engine was presented in a concrete form: a specific hop-bounded kernel K_r acting on a propensity-weighted graph, together with a collapse horizon W_{coh} that controls the recoverability of interference along the coherence axis t^-. For the purposes of BCQM IV we now step back and describe a *class* of microscopic graph-phase models that all share the same qualitative structure but may differ in local details. The universality claim is that, within this class, the emergent inertial dynamics and the associated inertial noise spectrum share a common scaling form once expressed in terms of W_{coh} and $m_{\text{eff}}(W_{\text{coh}})$.

We start from the primitives set out in the Primitives note:

- a countable set of potential Events $E \in \mathbb{E}$,
- a directed adjacency relation $R \subset \mathbb{E} \times \mathbb{E}$,
- complex edge amplitudes $a : R \to \mathbb{C}$ with $|a| \leq 1$,

and the hop-bounded selection rule. At a realised Event E_n the engine considers all candidate successors x reachable in at most r hops, constructs a local kernel

$$K_r(E_n \to x) \;=\; \sum_{\substack{\gamma: E_n \to x \\ \mathrm{len}(\gamma) \le r}} A[\gamma]\, F[\gamma; w], \tag{7}$$

where $A[\gamma]$ is the path amplitude (product of edge amplitudes) and $F[\gamma; w]$ is a weight functional depending on a local window parameter w satisfying $w \lesssim W_{\mathrm{coh}}$. The propensity for realising x as the next Event is proportional to $|K_r(E_n \to x)|^2$, and the q-wave is understood as the evolving propensity field derived from these kernels.

To formulate a universality statement we make the following structural assumptions about the microscopic rules:

1. **Locality and hop-boundedness.** The kernel at step n depends only on the finite neighbourhood of E_n inside the hop radius r and the window $w \lesssim W_{\mathrm{coh}}$; no long-range, ad hoc nonlocal terms are added.
2. **Stationarity and mild homogeneity.** On scales large compared to the microscopic Event spacing but small compared to macroscopic experimental scales, the statistics of the graph and the amplitudes are approximately stationary and statistically homogeneous in the emergent spacetime sense.
3. **Phase covariance.** The selection rule depends only on relative phases and moduli of the amplitudes in the local neighbourhood, and is invariant under global phase shifts of the q-wave.
4. **Finite coherence horizon.** There is a well-defined horizon W_{coh} beyond which advanced and retarded amplitude contributions no longer produce recoverable interference in the kernel; this horizon sets a physical time-scale for coherent correlations.

These assumptions are not derived from more primitive postulates; they define the universality class in which the present analysis is carried out. In particular, we are excluding long-range, fine-tuned nonlocal couplings and strongly inhomogeneous or non-stationary graph statistics on the scales of interest. If such effects are allowed, the simple scaling forms we propose for the inertial noise spectra may fail or acquire additional structure, and the universality claims of this section would have to be revisited.

Within this class, different microscopic choices of edge amplitudes, local graph architecture, or the detailed form of $F[\gamma; w]$ correspond to different “models”. The role of BCQM IV is not to favour a particular microscopic realisation, but to show that certain large-scale quantities—in particular, the inertial noise spectrum of realised trajectories—depend on the model only through a small set of dimensionless parameters, with the overall scaling fixed by W_{coh} and $m_{\mathrm{eff}}(W_{\mathrm{coh}})$.

3.2 Continuum limit and effective action

The companion BCQM technical note on emergent inertia and invariant action [5] develops the continuum-limit programme in detail. Here we summarise only the structural outcome and how it feeds into the inertial noise analysis.

Under the assumptions listed above, we *assume* that there exists a coarse-graining scale at which the realised Event graph can be embedded into an effective Lorentzian spacetime, and that the statistics of realised threads can be described by path weights of the form

$$\mathbb{P}[x(\cdot)] \;\propto\; \exp\Big(-\frac{1}{\hbar} S_{\mathrm{eff}}[x(\cdot)]\Big), \tag{8}$$

where S_{eff} is an effective action functional on smooth trajectories $x(\cdot)$, and $\hbar$ is used purely as a reference scale for phases. For a broad class of graph-phase models the leading part of this action in the emergent regime is the familiar relativistic form

$$S_{\text{eff}}[x(\cdot)] \;=\; -m_{\text{eff}} c \int \mathrm{d}s \;+\; S_{\text{sub}}[x(\cdot)], \tag{9}$$

where $\mathrm{d}s$ is the invariant line element of the emergent metric and S_{sub} contains subleading terms (higher derivatives, couplings to emergent fields, and possible memory kernels). The effective mass m_{eff} is determined by the same microscopic features that define the coherence horizon W_{coh}; BCQM III together with the BCQM technical note on emergent inertia and invariant action [5] show that, for the BCQM growth engine, m_{eff} scales as $m_{\text{eff}} \propto W_{\text{coh}}^{-2}$ up to model-dependent constants.

The Euler–Lagrange equations derived from S_{eff} describe the deterministic inertial drift of the realised trajectory in the emergent spacetime. However, because the underlying process is still a stochastic selection of Events guided by propensities, the actual realised paths deviate from these equations by small fluctuations. In the continuum description these deviations can be modelled as stochastic corrections to the effective equations of motion, for example as a Langevin-type equation

$$m_{\text{eff}} \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} \;+\; \frac{\delta S_{\text{sub}}}{\delta x_\mu} \;=\; \xi^\mu(\tau), \tag{10}$$

where τ is the proper time along the emergent trajectory and ξ^μ is an effective noise term summarising the residual stochasticity of the graph-phase process after coarse-graining.

The universality question is then shifted to the properties of the noise term $\xi^\mu(\tau)$: to what extent are its correlation functions fixed by the structural features of the graph-phase model class, and how do they depend on W_{coh}? In particular, the inertial noise observables defined in the previous section—the velocity and acceleration power spectra—are determined by the two-point function of ξ^μ along the worldline. Different microscopic models correspond to different detailed shapes of this two-point function at very short proper-time separations, but we will argue that once expressed in appropriate dimensionless variables the large-scale structure is universal.

3.3 Inertial noise kernels and scaling with W_{coh}

Let $\xi(t)$ denote the component of the effective noise along some fixed spatial direction in an inertial laboratory frame, after projecting the four-dimensional equation of motion onto that frame and coarse-graining to the timescales defined in the previous section. We assume that in the regime of interest $\xi(t)$ is a stationary process with zero mean and autocorrelation

$$C_\xi(\Delta t; W_{\text{coh}}) \;=\; \mathbb{E}\big[\xi(t)\,\xi(t+\Delta t)\big], \tag{11}$$

which may depend parametrically on the coherence horizon W_{coh}. The power spectral density of the noise is then

$$S_\xi(\omega; W_{\text{coh}}) \;=\; \int_{-\infty}^{\infty} C_\xi(\Delta t; W_{\text{coh}})\, e^{-i\omega\Delta t}\,\mathrm{d}\Delta t. \tag{12}$$

Because the inertial motion is governed by the effective mass $m_{\text{eff}}(W_{\text{coh}})$, and the only intrinsic time-scale entering the propensity correlations is set by the coherence horizon W_{coh}, basic dimensional and scaling considerations imply that the inertial noise spectra for velocity and acceleration can be written in the form

$$S_v(\omega; W_{\text{coh}}) = A_v(W_{\text{coh}})\, F_v(\omega W_{\text{coh}}), \tag{13}$$

$$S_a(\omega; W_{\text{coh}}) = A_a(W_{\text{coh}})\, F_a(\omega W_{\text{coh}}), \tag{14}$$

where F_v and F_a are dimensionless shape functions and $A_v(W_{\text{coh}})$, $A_a(W_{\text{coh}})$ are amplitude prefactors that carry the dimensions. The universality claim is that:

- the *functional forms* of F_v and F_a are the same for all microscopic graph-phase models in the class defined above (up to a small number of dimensionless parameters characterising local connectivity, mild anisotropy, and the shape of the short-range correlation kernel), and
- the *scaling* of A_v and A_a with W_{coh} is fixed once the relation $m_{\mathrm{eff}}(W_{\mathrm{coh}})$ is known, so that no further free exponents can be tuned.

In particular, given the BCQM III result that $m_{\mathrm{eff}} \propto W_{\mathrm{coh}}^{-2}$, one can express the amplitude prefactors in terms of m_{eff} and W_{coh}, for example in a schematic form

$$A_a(W_{\mathrm{coh}}) \sim \frac{1}{m_{\mathrm{eff}}^2} \frac{1}{W_{\mathrm{coh}}^{\beta}}, \tag{15}$$

with a fixed exponent β determined by the short-time behaviour of the effective noise kernel $C_\xi(\Delta t; W_{\mathrm{coh}})$. In practice β is controlled by how rapidly these correlations decay on the scale of W_{coh} and is expected to be an $\mathcal{O}(1)$ number for all models in the universality class defined above. The precise value of β, and the detailed shapes of F_v and F_a, are obtained by working through the continuum limit of the BCQM III engine, which we carry out in the BCQM technical note on emergent inertia and invariant action [5].

For the purposes of this paper it is enough at this point to record the following:

1. There exists an intrinsic inertial noise kernel encoded in $C_\xi(\Delta t; W_{\mathrm{coh}})$, even in the absence of ordinary environmental noise.
2. This kernel induces characteristic velocity and acceleration power spectra with a scaling form $S(\omega; W_{\mathrm{coh}}) = A(W_{\mathrm{coh}})F(\omega W_{\mathrm{coh}})$.
3. The dependence of $A(W_{\mathrm{coh}})$ on W_{coh} is tied to the effective mass $m_{\mathrm{eff}}(W_{\mathrm{coh}})$ derived in BCQM III, so that both inertia and inertial noise emerge from the same underlying graph-phase structure.

The remainder of the paper makes these statements precise for the BCQM growth engine and explores their numerical and observational consequences. At the same time, several aspects of the continuum-limit picture remain open. In particular, a fully rigorous derivation of the Lorentzian metric and invariant action from the graph-phase dynamics, and a precise characterisation of the universality class in which the scaling forms hold, are left to future work and to the programme outlined in the BCQM technical note [5]. Outside this class — for example, in strongly inhomogeneous or non-stationary regimes, or for models with explicit long-range nonlocality — the inertial noise spectrum may depart from the simple scaling forms proposed here.

4 Entangled cluster primitive and inertial noise

4.1 Configuration-space clusters

In BCQM II, entangled systems are described at the q-wave level by joint states on the tensor-product Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$, with the joint q-wave encoding propensities for correlated outcomes of measurements on the subsystems. At the level of BCQM Primitives and the BCQM III growth engine, the basic objects are single-particle threads and their event graphs. To accommodate entangled systems within this primitives language we introduce *entangled clusters*: collections of realised threads whose future event options are constrained by a single joint propensity structure, rather than by independent q-waves.

For a pair of subsystems A and B it is convenient to work in configuration space. The potential cluster Events are labelled by configuration points (x_A, x_B), with x_A and x_B denoting the coarse-grained positions (or more general configuration coordinates) of the two subsystems. We then define a configuration-space Event graph whose vertices are these joint configurations and whose edges encode allowed joint moves of the cluster. The simplest construction starts from the product graph of the individual event graphs and then restricts to a subgraph compatible with the joint q-wave on $\mathcal{H}_A \otimes \mathcal{H}_B$:

- the *product graph* has vertices (x_A, x_B) for all combinations of vertices x_A on the A-graph and x_B on the B-graph, with edges induced by the individual adjacency relations;
- the *entangled cluster graph* is a subgraph of this product graph obtained by discarding vertices and edges that carry negligible joint propensity under the BCQM II joint q-wave.

In practice this restriction is implemented by evaluating the BCQM II joint q-wave in a configuration basis labelled by (x_A, x_B) and retaining only those configurations for which the corresponding joint propensity is non-negligible on the timescales of interest. The result is a configuration-space graph on which the BCQM growth engine can act in exactly the same hop-bounded manner as before, but now with Events corresponding to joint configurations.

The cluster-level kernel for a candidate successor (x'_A, x'_B) given a current realised cluster Event (x_A, x_B) takes the schematic form

$$K_r^{(AB)}((x_A, x_B) \to (x'_A, x'_B)) \;=\; \sum_{\substack{\gamma:(x_A,x_B)\to(x'_A,x'_B) \\ \mathrm{len}(\gamma)\le r}} A_{AB}[\gamma]\, F_{AB}[\gamma; w], \tag{16}$$

with $A_{AB}[\gamma]$ the joint path amplitude on the cluster graph and $F_{AB}[\gamma; w]$ a window functional respecting the same coherence horizon condition $w \lesssim W_{\mathrm{coh}}$ as in the single-thread case. The propensity for the cluster to realise (x'_A, x'_B) at the next step is then proportional to $|K_r^{(AB)}((x_A, x_B) \to (x'_A, x'_B))|^2$. By construction this setup reproduces, at the level of realised-event frequencies, the BCQM II joint q-wave law on $\mathcal{H}_A \otimes \mathcal{H}_B$ when one projects back to subsystem outcomes.

4.2 Locality, no-signalling, and conservation

A key requirement for the entangled-cluster primitive is that it should respect locality, no-signalling, and exact conservation laws at the event level. In the configuration-space graph these requirements take the following operational form:

- **Hop-bounded locality in configuration space.** At each step the BCQM growth engine only considers joint moves within a finite hop radius r on the cluster graph. This means that the next cluster Event depends only on a finite neighbourhood of (x_A, x_B) in configuration space and on the local window $w \lesssim W_{\mathrm{coh}}$, with no ad hoc long-range couplings added.
- **Exact conservation of global charges.** Quantities such as total momentum, total charge, or total spin are encoded as constraints on the allowed edges of the cluster graph. Only edges $(x_A, x_B) \to (x'_A, x'_B)$ consistent with these conservation laws are present, so every realised Event automatically respects Q_{tot} conservation.
- **No-signalling.** Local interventions on subsystem A that are confined to a spacetime region outside the past light cone of B are represented as local modifications of the edges and amplitudes associated with the A-coordinates in the cluster graph. Because the cluster kernel remains hop-bounded and phase-covariant, and because the BCQM selection rule uses only local information within r and w, the marginal statistics for B obtained by summing over A-configurations are unaffected by such interventions. This is the configuration-space counterpart of the no-signalling proofs given in BCQM II.

Within this entangled-cluster primitive the centre-of-mass motion of the cluster can be analysed in the same way as a single realised thread in BCQM III. One defines coarse-grained centre-of-mass coordinates $X_{\rm cm}$ and velocities $V_{\rm cm}(t)$ from the sequence of realised cluster Events $(x_A(t), x_B(t))$, and then decomposes $V_{\rm cm}(t)$ into an ensemble-averaged drift plus fluctuations. The deterministic drift is governed by the same effective relativistic action as in the single-thread case, with an effective mass that scales with $W_{\rm coh}$ according to the BCQM III and technical-note analysis. The residual fluctuations define the *cluster inertial noise* spectrum, which receives contributions from the intrinsic stochasticity of the cluster graph as well as from possible internal degrees of freedom within the cluster.

For tightly bound clusters the internal degrees of freedom are rapidly relaxing, and on long timescales the inertial noise of $X_{\rm cm}$ approaches that of a single effective particle with mass $M_{\rm eff}$ and coherence horizon $W_{\rm coh}^{(\rm cm)}$ determined by the cluster as a whole. For more weakly bound or partially measured clusters there can be additional noise channels associated with internal rearrangements; these appear as extra structure in the centre-of-mass noise kernel but do not alter the basic scaling of the noise spectrum with the relevant coherence horizon. BCQM IV focuses on the tightly bound regime, where the entangled cluster behaves, for the purposes of inertial noise, like a single effective degree of freedom at the centre of mass, and leaves a full classification of cluster noise channels to future work.

5 Numerical experiments and observational prospects

5.1 Numerical estimation of the inertial noise spectrum

The first line of investigation is numerical. The BCQM III growth engine already provides a stochastic process for generating realised trajectories from a specified hop radius r, coherence horizon $W_{\rm coh}$, and choice of local graph-phase rule. To estimate the inertial noise spectrum we proceed as follows.

1. Fix a choice of microscopic graph-phase model within the class described earlier (local, hop-bounded, phase-covariant, finite $W_{\rm coh}$). For each parameter set, generate many independent realisations of a single-thread trajectory using the BCQM III engine.
2. For each trajectory, construct coarse-grained position and velocity time series $x(t)$ and $v(t)$ in the emergent laboratory frame, using the same discretisation conventions as in BCQM III.[1]
3. Subtract the ensemble-averaged drift to obtain velocity fluctuations $\delta v(t)$, and estimate the power spectral density $S_v(\omega)$ (and, if desired, the acceleration spectrum $S_a(\omega)$) using standard periodogram methods with appropriate windowing and averaging.
4. Repeat the procedure for a range of coherence horizons $W_{\rm coh}$ and other microscopic parameters, and fit the resulting spectra to the scaling forms $S_v(\omega; W_{\rm coh}) = A_v(W_{\rm coh})F_v(\omega W_{\rm coh})$ and $S_a(\omega; W_{\rm coh}) = A_a(W_{\rm coh})F_a(\omega W_{\rm coh})$ proposed earlier, extracting the amplitude scaling exponents and checking whether the shape functions F_v and F_a collapse across the model class.

A second numerical task is to extend this analysis to entangled clusters using the configuration-space primitive introduced in section 4. Here one generates realised cluster trajectories $(x_A(t), x_B(t))$, constructs centre-of-mass variables $X_{\rm cm}(t)$ and $V_{\rm cm}(t)$, and estimates the centre-of-mass inertial noise spectrum. By comparing tightly bound and more weakly bound cluster

[1] In practice the time series will be defined on a discrete grid with time step Δt large compared to the inter-Event spacing but small compared to the macroscopic scales of interest.

models one can test whether this centre-of-mass reduction holds in the tightly bound regime and identify additional noise channels associated with internal rearrangements.

5.2 Possible observational signatures

The second line of investigation is experimental or observational. The inertial noise predicted by BCQM manifests as an irreducible stochastic floor in the motion of systems whose external environment has been carefully controlled. In practice one would look for a characteristic power spectrum of residual acceleration noise that:

- persists as ordinary thermal, technical, and environmental noise sources are progressively reduced, and
- exhibits the scaling behaviour with W_{coh} and effective mass suggested by the numerical studies and the effective-action analysis.

Several broad classes of systems suggest themselves as potential laboratory or astrophysical probes:

- *High-Q mechanical or optomechanical oscillators,* cooled close to their ground state, where extremely small force noise can be resolved. Here BCQM inertial noise would appear as a residual acceleration noise floor after known noise sources are modelled and subtracted.
- *Cold-atom interferometers and matter-wave experiments,* where phase noise and trajectory fluctuations can be measured with high precision. In such setups BCQM inertial noise could imprint itself on the fringe visibility and on inferred acceleration statistics.
- *Precision inertial sensors* (for example, drag-free test masses in space missions), where long-time acceleration noise spectra are already a key diagnostic. A BCQM-induced inertial noise floor would contribute a specific spectral component that does not scale away with improved shielding and environmental control.

At the present stage of the BCQM programme it would be premature to tie these suggestions to a specific experimental proposal. BCQM IV restricts itself to defining the inertial noise observable, deriving its scaling structure within the BCQM framework, and outlining the type of numerical and experimental studies that would be needed to test the prediction. Later work will refine these estimates, incorporate realistic models of environmental noise and decoherence, and identify concrete parameter regimes in which the BCQM inertial noise floor might be observable.

6 Conclusion and outlook

The BCQM framework treats the q-wave as an informational field that constrains a single realised particle thread along the chronological axis t^+, while a finite coherence horizon W_{coh} encodes the recoverability of interference along the coherence axis t^-. BCQM III showed that, in the emergent spacetime regime, the average motion of a realised thread obeys an effective inertial law with a mass parameter $m_{\text{eff}}(W_{\text{coh}})$, and that this inertial response is tied to the same structure that defines the collapse horizon.

BCQM IV adds a further layer by identifying and characterising *inertial noise*: the irreducible fluctuations of realised trajectories around the effective inertial drift that remain even when ordinary environmental noise is idealised away. Within a broad class of local, hop-bounded graph-phase models with finite coherence horizon, we have argued that the velocity and acceleration noise spectra admit scaling forms of the type $S(\omega; W_{\text{coh}}) = A(W_{\text{coh}})\, F(\omega W_{\text{coh}})$, with the shape functions F largely universal across the model class and the amplitude scaling fixed once $m_{\text{eff}}(W_{\text{coh}})$ is known. In this sense the existence and structure of an inertial noise floor is a falsifiable consequence of the BCQM primitives and the BCQM III growth engine, rather than an adjustable phenomenological ingredient.

We have also extended the BCQM primitives to *entangled clusters* by introducing a configuration-space event graph that reproduces the BCQM II joint q-wave law on $\mathcal{H}_A \otimes \mathcal{H}_B$ while preserving locality, exact conservation laws, and no-signalling at the event level. Within this primitive the centre-of-mass motion of a tightly bound cluster behaves, for the purposes of inertial noise, like that of a single effective particle with its own coherence horizon and effective mass. This provides the bridge needed to connect the single thread analysis to potential experimental probes, where the relevant degrees of freedom are typically collective modes of many-body systems.

Several open questions and directions for further work remain. On the conceptual side, it would be valuable to put the universality arguments for the inertial noise spectrum on a more rigorous mathematical footing, with explicit conditions under which a Lorentzian metric and invariant action emerge from the underlying graph-phase dynamics. The role of dimensionality selection, and the extent to which the BCQM primitives privilege three spatial dimensions in the emergent regime, also deserve closer examination. On the phenomenological side, a more detailed analysis of candidate experimental platforms — including realistic models of environmental noise and decoherence — is needed to assess whether the inertial noise floor predicted by BCQM could be observed in practice.

Beyond the present paper, a dedicated follow-up study will implement the scaling analysis numerically, provide order-of-magnitude estimates for the inertial noise floor in realistic units, and develop explicit configuration-space cluster graphs as worked examples of the entangled cluster primitive introduced here.

Finally, BCQM V will extend the present analysis to include gravity, treating spacetime curvature and stochastic gravitational effects as emergent manifestations of the same event-graph structure that underlies collapse, inertia, and inertial noise. In that broader context the results of BCQM IV should be viewed as a first step towards a unified description of quantum events, inertia, and spacetime geometry within the boundary-condition framework.

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