BCQM Technical Note: Emergent Inertia and Invariant Action from Graph-Phase Dynamics

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Abstract

This technical note consolidates two ingredients underpinning the Boundary-Condition Quantum Mechanics (BCQM) programme. First, we derive the effective inertial response of a moving quantum system by formulating its open-system dynamics in proper time, obtaining the velocity dependence solely from the transformed environmental spectrum, and adiabatically eliminating a fast internal phase. The result reproduces the scaling

$$m_{\rm eff} \;\sim\; I_{\rm eff}\,\omega_0^2/c^2$$

and identifies corrections, clarifying how inertia emerges as a feedback effect between proper-time ticking and stage-time kinematics. Second, we outline a continuum-limit programme for BCQM event-graph dynamics in which local, hop-bounded graph-phase rules give rise to an effective relativistic action for realised worldlines. This provides the structural link between the collapse horizon, emergent inertia, and the inertial noise spectrum investigated in BCQM IV.

1 Introduction and overview

The BCQM framework treats the q-wave as an informational field that constrains a single realised particle thread along the chronological axis t^+, while a finite coherence horizon $W_{\rm coh}$ encodes the recoverability of interference along the coherence axis t^-. BCQM III shows that, in the appropriate emergent regime, the average motion of a realised thread obeys an effective inertial law with a mass parameter $m_{\rm eff}(W_{\rm coh})$ determined by the same structure that defines the collapse horizon. The present note collects and slightly expands the technical steps behind two related claims:

(i) Inertia can be derived from proper-time GKLS dynamics of an internal "clock" degree of freedom weakly coupled to an environment, with velocity dependence entering only through the transformed bath spectrum.

(ii) For a broad class of local, hop-bounded graph-phase rules, the coarse-grained statistics of realised event threads admit a description in terms of an effective relativistic action, with $m_{\rm eff}(W_{\rm coh})$ providing the inertial parameter in that action.

Section 2 presents the proper-time GKLS derivation of emergent inertia, originally developed as an internal Appendix for BCQM III[1]. Section 3 then gives a programme-level outline for obtaining an invariant action from graph-phase dynamics in the Primitives model, together with a brief discussion of noise and fluctuations. Both sections are intended as technical background for BCQM III [1] and BCQM IV, rather than as self-contained experimental proposals.

Throughout this note we work in units where c is the speed of light and $\hbar$ appears explicitly when convenient.

2 Emergent inertia from proper-time GKLS dynamics

2.1 Statement of the result

Consider a two-level "clock" degree of freedom (internal phase) rigidly attached to a body with bare mass m_b, interacting weakly with an environment. After adiabatic elimination of the fast internal phase, the effective inertial mass in the laboratory (Stage) frame takes the form

$$m_{\text{eff}} \;=\; m_b \;+\; I_{\text{eff}}\,\frac{\omega_0^2}{c^2} \;+\; I_{\text{eff}}\,\frac{\omega_0\,\delta\omega^{(2)}}{c^2} \;+\; \mathcal{O}\!\left(\frac{v^4}{c^4}\right), \tag{1}$$

where I_{eff} is an emergent inertia parameter of the internal clock, ω_0 is its bare splitting, and $\delta\omega^{(2)}$ is the even-in-v quadratic correction from the environment-induced Lamb shift in proper time. The explicit $\mathcal{O}(v^4/c^4)$ reminder indicates that all v^2/c^2 contributions have been displayed, in contrast to earlier shorthand that wrote $\mathcal{O}(v^2/c^2)$.

In the BCQM interpretation, the internal two-level system is a coarse-grained representation of quantum ticking along the coherence axis t^-; its phase plays the role of the actor time. Collapse is implemented by CPTP damping channels, and the environment determines the proper-time rates and Lamb shifts via its correlation spectrum sampled along the worldline, not by any ad hoc velocity dependence. Inertia then emerges as a derived property of the actor–environment coupling.

2.2 Proper-time GKLS generator

We write the most general phase-covariant Markovian dynamics for the qubit in its own proper time τ as

$$\frac{\mathrm{d}\rho}{\mathrm{d}\tau} \;=\; \mathcal{L}^{(\tau)}[\rho] \;=\; -\frac{\mathrm{i}}{\hbar}\Big[\frac{\hbar}{2}\,\omega_{\text{eff}}^{(\tau)}(v)\,\sigma_z,\,\rho\Big] + \gamma_+^{(0)}(v)\,\mathcal{D}[\sigma^+]\rho + \gamma_-^{(0)}(v)\,\mathcal{D}[\sigma^-]\rho + \gamma_\phi^{(0)}(v)\,\mathcal{D}[\sigma_z]\rho, \tag{2}$$

with the usual dissipator

$$\mathcal{D}[A]\rho \;=\; A\rho A^\dagger - \tfrac{1}{2}\{A^\dagger A,\,\rho\}. \tag{3}$$

The effective proper-time frequency $\omega_{\text{eff}}^{(\tau)}(v) = \omega_0 + \delta\omega(v)$ includes the Lamb shift from the environment. All rates $\gamma_\bullet^{(0)}(v)$ are per unit proper time.

Phase covariance here means that the generator commutes with rotations about the z axis in the Bloch sphere, so that the dynamics preserves the structure of energy eigenstates and acts only on populations and the off-diagonal phase.

2.3 Environment spectrum and rates

Velocity dependence is not imposed by hand but derived from how a moving system samples the environment's correlation spectrum. In the Davies weak-coupling limit the proper-time rates are proportional to an effective spectrum $S_{\text{eff}}^{(\tau)}(\omega;v)$ evaluated at $\pm\omega_0$:

$$\gamma_-^{(0)}(v) \propto S_{\text{eff}}^{(\tau)}(+\omega_0;v), \tag{4}$$

$$\gamma_+^{(0)}(v) \propto S_{\text{eff}}^{(\tau)}(-\omega_0;v), \tag{5}$$

and the Lamb shift $\delta\omega(v)$ is obtained from a Kramers–Kronig transform of $S^{(\tau)}_{\rm eff}(\omega; v)$. For a thermal environment the KMS condition holds in the co-moving frame:

$$\frac{\gamma_+^{(0)}(v)}{\gamma_-^{(0)}(v)} \;=\; \exp\!\left[-\frac{\hbar\omega_0}{k_{\rm B}\,T_{\rm eff}(v)}\right], \tag{6}$$

where $T_{\rm eff}(v)$ captures Doppler and aberration effects of the background field as seen along the worldline. No explicit model for $T_{\rm eff}(v)$ is needed for the inertial scaling; it is enough that the spectrum transforms covariantly under changes of the worldline.

2.4 From proper time to laboratory time

Let θ denote the internal phase of the clock. In proper time its evolution is

$$\frac{{\rm d}\theta}{{\rm d}\tau} \;=\; -\omega^{(\tau)}_{\rm eff}(v). \tag{7}$$

Transforming to Stage (laboratory) time via ${\rm d}t = \gamma(v)\,{\rm d}\tau$, with $\gamma(v)$ the usual Lorentz factor, yields the target phase rate

$$\dot\theta_{\rm ss}(v) \;\equiv\; \frac{{\rm d}\theta}{{\rm d}t} \;=\; -\frac{\omega^{(\tau)}_{\rm eff}(v)}{\gamma(v)}. \tag{8}$$

Expanding for small $\beta = v/c$ with $\gamma(\beta) \simeq 1 + \frac{1}{2}\beta^2$ and $\omega^{(\tau)}_{\rm eff}(v) \simeq \omega_0 + \delta\omega^{(2)}\beta^2$ gives

$$\dot\theta_{\rm ss}(v) \;\simeq\; -\omega_0 + \left(\frac{\omega_0}{2} - \delta\omega^{(2)}\right)\frac{v^2}{c^2} + \mathcal{O}\!\left(\frac{v^4}{c^4}\right), \tag{9}$$

Here $\delta\omega^{(2)}$ denotes the coefficient of the β^2 term in the small-velocity expansion of $\omega^{(\tau)}_{\rm eff}(v)$, so that $\delta\omega^{(2)} = \frac{1}{2}(\partial^2\omega^{(\tau)}_{\rm eff}/\partial\beta^2)|_{\beta=0}$ and $\delta\omega^{(2)}$ vanishes at $v = 0$. which is manifestly even in v, as required for contributions to the kinetic energy.

2.5 Adiabatic elimination and effective Lagrangian

Treat the phase θ as a fast variable that relaxes towards $\dot\theta_{\rm ss}(v)$ with a rate λ set by the dissipator in equation (2). Standard projection methods (such as the Mori–Zwanzig formalism) then yield, for slow kinematics $x(t)$, an effective Lagrangian contribution of the form

$$L_{\rm eff}[x,\dot x] \;\supset\; \frac{1}{2}\,m_{\rm eff}\,\dot x^2 \qquad \text{with} \qquad m_{\rm eff} \;\propto\; \left(\frac{\partial\dot\theta_{\rm ss}}{\partial v}\right)^2\bigg|_{v=0}\,\frac{I_{\rm eff}}{\Lambda}, \tag{10}$$

where $I_{\rm eff}$ and Λ are positive constants determined by the microdynamics (coupling strengths, spectral densities, and the details of phase locking). Finite relaxation introduces a small "jerk" correction in the Euler–Lagrange equation:

$$F \;=\; m_{\rm eff}a + \tau\,m_{\rm eff}\,\dot a + \cdots\,, \qquad \tau \simeq \lambda^{-1}, \tag{11}$$

so that τ is the phase-locking time of the internal clock. Combining equation (9) and equation (10) recovers the scaling form equation (1), with explicit expressions for the coefficients in terms of spectral and coupling data.

2.6 Interpretation within BCQM

In BCQM terms:

- The internal two-level clock is a coarse-grained representation of quantum ticking along t^-; its phase plays the role of the actor time governing coherence.
- Collapse is implemented via CPTP damping channels whose rates and Lamb shifts are determined by $S^{(\tau)}_{\text{eff}}(\omega; v)$, not by arbitrary velocity-dependent terms.
- The actor–environment feedback loop is the operational content behind inertia: attempts to change v drive $\dot{\theta}$ away from its target value, and the coupled system resists that change.

Thus inertia arises as a derived property of the actor–environment coupling rather than as a primitive intrinsic quantity. At the BCQM level this provides the link between a finite coherence horizon W_{coh} and the effective inertial response, justifying the scaling $m_{\text{eff}}(W_{\text{coh}}) \propto W_{\text{coh}}^{-2}$ used in BCQM III and BCQM IV.

Remark 1. *The identification of emergent inertia with feedback between proper-time ticking and stage-time dynamics is compatible with the equivalence principle: the same collapse-tick mechanism that resists acceleration locally also sources spacetime curvature in aggregate. In this sense inertial and gravitational mass share a common origin within the BCQM framework, although a full treatment of gravity is deferred to later work.*

3 From graph-phase dynamics to an invariant action

3.1 Graph-phase setup and transfer operator

We now turn to the discrete event-graph level of the Primitives model. The basic ingredients are:

- a countable set of potential Events $E \in \mathbb{E}$;
- a directed adjacency relation $R \subset \mathbb{E} \times \mathbb{E}$;
- complex edge amplitudes $a : R \to \mathbb{C}$ with $|a| \leq 1$;
- a hop radius r defining the local neighbourhood; and
- a window parameter w satisfying $w \lesssim W_{\text{coh}}$ that bounds coherent contributions along t^-.

At a realised Event E_n the BCQM growth engine evaluates a local kernel K_r over successors x reachable within r hops:

$$K_r(E_n \to x) \;=\; \sum_{\substack{\gamma: E_n \to x \\ \text{len}(\gamma) \leq r}} A[\gamma]\, F[\gamma; w], \tag{12}$$

where $A[\gamma] = \prod_{e \in \gamma} a(e)$ is the path amplitude obtained by multiplying the edge amplitudes along γ, and $F[\gamma; w]$ is a real window functional that suppresses contributions from segments lying outside the hop-bounded, finite-coherence neighbourhood (in particular $F[\gamma; w] \to 0$ when $\text{len}(\gamma)$ exceeds the effective window $w \lesssim W_{\text{coh}}$). The propensity for realising x as the next Event is proportional to $|K_r(E_n \to x)|^2$.

To formulate a universality statement we consider a class of such graph-phase models satisfying:

1. *Locality and hop-boundedness:* the kernel depends only on the finite neighbourhood of E_n within r and w.
2. *Stationarity and mild homogeneity:* on scales large compared to the microscopic Event spacing but small compared to macroscopic scales, the statistics of the graph and amplitudes are approximately stationary and homogeneous in the emergent sense.

3. *Phase covariance:* the selection rule depends only on relative phases and moduli in the local neighbourhood and is invariant under global phase shifts of the q-wave.
4. *Finite coherence horizon:* a well-defined $W_{\rm coh}$ bounds the time extent of recoverable interference.

Within this class, one can regard each step of the growth engine as a transfer operator acting on coarse-grained variables (position and possibly momentum bias). Repeated application of this operator admits a Kramers–Moyal or central-limit expansion that leads to a continuum description.

3.2 Coarse-graining and emergent dispersion

Introduce a coarse-graining scale at which the realised Event graph can be embedded into an effective spacetime and the statistics of realised threads can be described by probability amplitudes $\psi(x,t)$ or path weights on trajectories. In Fourier space, the transfer operator induces a dispersion relation for dominant modes of the form

$$\omega^2 \;\simeq\; c^2\,\mathbf{k}^2 + m_{\rm eff}^2 c^4/\hbar^2 + \text{subleading terms}, \tag{13}$$

where the effective mass $m_{\rm eff}$ is determined by the same microscopic parameters that control $W_{\rm coh}$. The precise derivation depends on the detailed step statistics, but the key structural feature is the emergence of a Lorentz-like quadratic form in $(\omega, \mathbf{k})$ for long-wavelength modes.

In configuration space this dispersion relation corresponds to an effective quadratic form on velocities, and hence to an action functional on smooth trajectories $x(\cdot)$.

3.3 Emergent metric and effective action

Under the assumptions above there exists an emergent metric $g_{\mu\nu}$ and an effective action of the form

$$S_{\rm eff}[x(\cdot)] \;=\; -\,m_{\rm eff}c\int {\rm d}s + S_{\rm sub}[x(\cdot)], \tag{14}$$

where ${\rm d}s^2 = g_{\mu\nu}\,{\rm d}x^\mu {\rm d}x^\nu$, and $S_{\rm sub}$ collects subleading terms, such as higher-derivative corrections to the kinetic term, couplings to emergent background fields (for example curvature couplings schematically of the form $\int R\,{\rm d}s$), spin–orbit interactions for internal degrees of freedom, and weak non-Markovian memory kernels. The path weight for a realised trajectory can then be written schematically as

$$\mathbb{P}[x(\cdot)] \;\propto\; \exp\!\Big(-\frac{1}{\hbar}\,S_{\rm eff}[x(\cdot)]\Big). \tag{15}$$

The Euler–Lagrange equations from $S_{\rm eff}$ describe the deterministic drift of the realised trajectory in the emergent spacetime and identify $m_{\rm eff}$ as the inertial parameter. BCQM III and the preceding section show that $m_{\rm eff}$ also emerges from proper-time GKLS dynamics with a scaling $m_{\rm eff}(W_{\rm coh}) \propto W_{\rm coh}^{-2}$, thus tying together the collapse horizon and the invariant action.

3.4 Noise, fluctuations, and inertial noise

Because the underlying graph-phase process is stochastic, realised trajectories deviate from the deterministic paths defined by $S_{\rm eff}$. At the continuum level these deviations can be modelled as stochastic corrections to the effective equations of motion. In a simple Langevin picture one writes

$$m_{\rm eff}\,\frac{{\rm d}^2 x^\mu}{{\rm d}\tau^2} + \frac{\delta S_{\rm sub}}{\delta x_\mu} \;=\; \xi^\mu(\tau), \tag{16}$$

where τ is proper time along the emergent trajectory and ξ^μ is an effective noise term summarising residual stochasticity after coarse-graining.

The statistics of ξ^μ — in particular its two-point function along the worldline — determine the inertial noise observables of BCQM IV: the power spectral densities of velocity and acceleration fluctuations. Different microscopic graph-phase models correspond to different detailed shapes of the noise kernel at very short proper-time separations, but once expressed in suitable dimensionless variables the large-scale features are expected to be universal across the model class.

3.5 Summary and outlook

This section has outlined, at a programme level, how hop-bounded, phase-covariant graph-phase dynamics with a finite coherence horizon can give rise to an effective relativistic action for realised trajectories, together with an emergent inertial parameter $m_{\text{eff}}(W_{\text{coh}})$. Combined with the proper-time GKLS derivation of section 2, this justifies treating both the deterministic drift and the inertial noise of BCQM realised threads within an effective action plus noise framework.

A fully rigorous derivation of the continuum limit, including precise conditions for the emergence of a Lorentzian metric and control over subleading terms in S_{sub}, remains an open mathematical problem and is left for future work. For the purposes of the BCQM III and BCQM IV phenomenology, however, the structural link between W_{coh}, $m_{\text{eff}}(W_{\text{coh}})$, and the inertial noise spectrum is the key output.

Beyond the present note, several natural directions for further work suggest themselves. These include:

- establishing rigorous continuum-limit theorems for broad classes of hop-bounded graph-phase models, including precise conditions for the emergence of a Lorentzian metric;
- working out explicit examples of the graph-to-dispersion correspondence on simple lattices, to make the link between the discrete transfer operator and the effective action more concrete;
- extending the present framework to curved emergent backgrounds, in which the coarse-grained metric departs from Minkowski form and the role of S_{sub} in sourcing curvature can be analysed systematically;
- computing noise kernels and their scaling properties in more detail, including numerical studies of universality across different microscopic graph ensembles;
- developing an operator-algebraic formulation of the transfer operator and its spectrum, to place the stochastic growth picture on a firmer mathematical footing.